Energy storage via high-energy density composite flywheels

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Presentation overview

- Introduction - Motivation - Basic Principle

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- Stacked-ply disk design problem

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- Conclusion

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- Gasoline: 14000 Wh/kg
- Hydrogen: 38000 Wh/kg

Flywheel Energy Storage (FES)

- FES usage
  - Electrical load leveling
  - Batteries for electrical vehicles
  - Pulsed power supplies
- FES design challenges
  - Bearings
  - Drive/generator
  - Containment
  - Flywheel

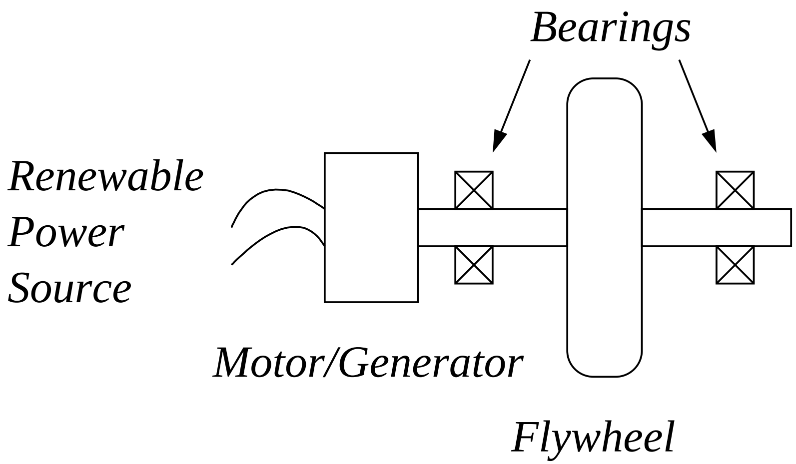
Flywheel Energy Storage - Basic ideas

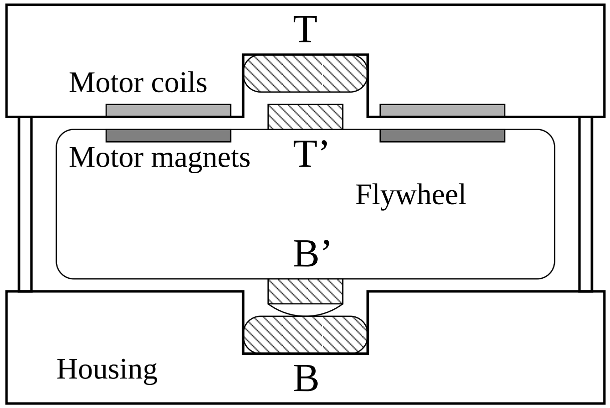
- A kinetic energy storage device
- Maximum energy density:

$$\alpha = \frac{1}{2}\left(I\omega_f^2\right)\left(\frac{1}{3600\rho V}\right) \text{ Wh/kg}$$

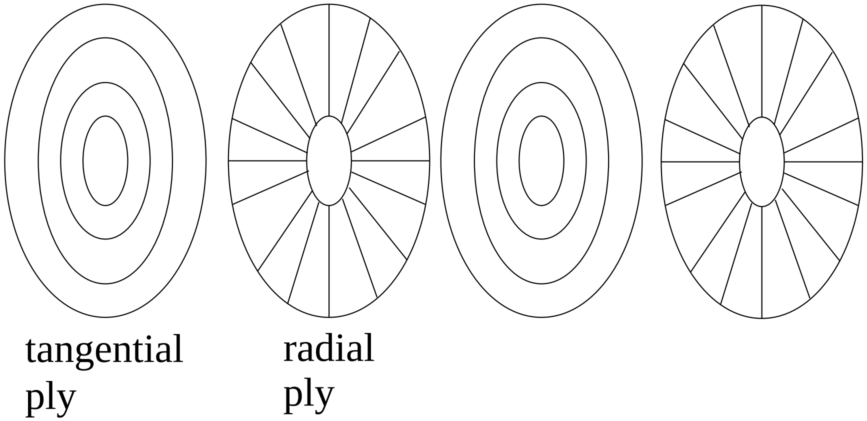
I - moment of inertia of the disk (kg-m^2),
ω_f - failure speed of the disk (rad/s),
ρ - density (kg/m^3),
V - volume (m^3).

Flywheel Energy Storage - Schematics



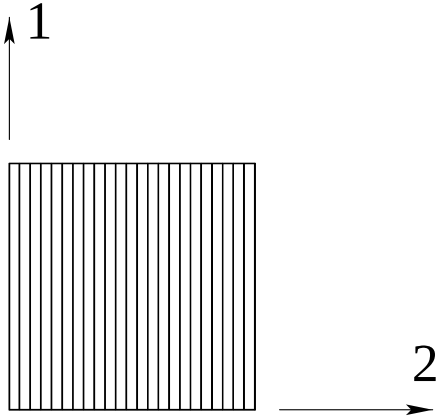


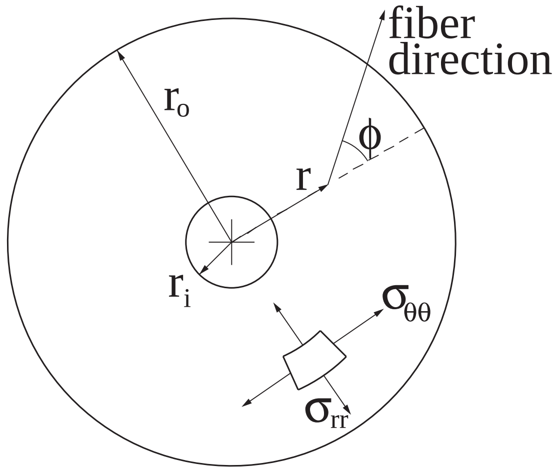
Stacked-ply Flywheel



- Alternate layers of radial and tangential plies
- Can fiber angle variation improve performance?
- How can such disks be constructed?

Constitutive relationship





Ply stiffness relationship

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{pmatrix}_k = \underbrace{\begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix}_k}_{[\mathbf{Q}]_k} \begin{pmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \end{pmatrix}_k$$

The k-th ply stiffness

$$\begin{pmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \tau_{r\theta} \end{pmatrix}_k = \underbrace{\begin{pmatrix} \bar{Q}_{rr} & \bar{Q}_{r\theta} & \bar{Q}_{r6} \\ \bar{Q}_{r\theta} & \bar{Q}_{\theta\theta} & \bar{Q}_{\theta 6} \\ \bar{Q}_{r6} & \bar{Q}_{\theta 6} & \bar{Q}_{66} \end{pmatrix}_k}_{[\bar{\mathbf{Q}}(\phi)]_k} \begin{pmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \gamma_{r\theta} \end{pmatrix}_k$$

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Stiffness components

$$
\begin{aligned}
\bar{Q}_{rr} &= Q_{11}\cos^4\phi + Q_{22}\sin^4\phi + (2Q_{12}+4Q_{66})\cos^2\phi\sin^2\phi \\
\bar{Q}_{r\theta} &= Q_{12}(\cos^4\phi + \sin^4\phi) + (Q_{11}+Q_{22}-4Q_{66})\cos^2\phi\sin^2\phi \\
\bar{Q}_{22} &= Q_{11}\sin^4\phi + Q_{22}\cos^4\phi + (2Q_{12}+4Q_{66})\cos^2\phi\sin^2\phi \\
\bar{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\cos^2\phi\sin^2\phi \\
&\quad + Q_{66}(\cos^4\phi+\sin^4\phi) \\
\bar{Q}_{r6} &= (Q_{11}-Q_{12}-2Q_{66})\cos^3\phi\sin\phi \\
&\quad - (Q_{22}-Q_{12}-2Q_{66})\cos\phi\sin^3\phi \\
\bar{Q}_{\theta 6} &= (Q_{11}-Q_{12}-2Q_{66})\cos\phi\sin^3\phi \\
&\quad - (Q_{22}-Q_{12}-2Q_{66})\cos^3\phi\sin\phi.
\end{aligned}
$$

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Ply properties

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = Q_{21} = \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

Laminate constitutive relations

$$\begin{pmatrix} \tilde{\sigma}_{rr} \\ \tilde{\sigma}_{\theta\theta} \\ \tilde{\tau}_{r\theta} \end{pmatrix} = [\mathbf{A}(\phi)] \begin{pmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \gamma_{r\theta} \end{pmatrix}$$

$$[\mathbf{A}(\phi)] = \lambda \underbrace{[\bar{\mathbf{Q}}(90^\circ)]}_{\text{tang. stiffness}} + (1-\lambda) \underbrace{\left(\frac{1}{2}[\bar{\mathbf{Q}}(\phi)] + \frac{1}{2}[\bar{\mathbf{Q}}(-\phi)] \right)}_{\text{radial stiffness}}$$

$$\lambda = \frac{\text{total thickness of tangential reinforcement plies}}{\text{total thickness of laminate}}.$$

Laminate constitutive relations (cont.)

Effective strain-stress

$$\begin{pmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \gamma_{r\theta} \end{pmatrix} = \underbrace{\begin{pmatrix} S_{rr} & S_{r\theta} & 0 \\ S_{r\theta} & S_{\theta\theta} & 0 \\ 0 & 0 & S_{66} \end{pmatrix}}_{[\mathbf{S}(\phi)]} \begin{pmatrix} \tilde{\sigma}_{rr} \\ \tilde{\sigma}_{\theta\theta} \\ 0 \end{pmatrix}$$

where $[\mathbf{S}(\phi)] = [\mathbf{A}(\phi)]^{-1}$ is the effective compliance matrix

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{pmatrix}_k = [\mathbf{T}(\phi)]_k [\bar{\mathbf{Q}}(\phi)]_k [\mathbf{S}(\phi)] \begin{pmatrix} \tilde{\sigma}_{rr} \\ \tilde{\sigma}_{\theta\theta} \\ 0 \end{pmatrix} .$$

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Coordinate trasformation

$$\mathbf{T}(\phi) = \begin{pmatrix} \cos^2\phi & \sin^2\phi & 2\cos\phi\sin\phi \\ \sin^2\phi & \cos^2\phi & -2\cos\phi\sin\phi \\ -\cos\phi\sin\phi & \cos\phi\sin\phi & \cos^2\phi\sin^2\phi \end{pmatrix}.$$

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Stress state

- Equilibrium: $\frac{d}{dr}(r\tilde{\sigma}_{rr}) - \tilde{\sigma}_{\theta\theta} + \rho\omega^2 r^2 = 0$

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- Compatibility: $r\frac{d\epsilon_{\theta\theta}}{dr} + \epsilon_{\theta\theta} - \epsilon_{rr} = 0$
- States:

$$x_1 = \frac{\tilde{\sigma}_{rr}}{\rho\omega^2 r_o^2}, \quad x_2 = \frac{\tilde{\sigma}_{\theta\theta}}{\rho\omega^2 r_o^2}, \quad x_3 = \phi, \quad x_4 = r/r_o = \tau$$

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- Control variable: $\Longrightarrow$ derivative of fiber angle: $u = d\phi/d\tau$

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- Control variable: $\Longrightarrow$ derivative of fiber angle: $u = d\phi/d\tau$
- Nondimensional compliance: $\mathcal{S}_{\theta\theta} = E_{11}S_{\theta\theta}$ and $\mathcal{S}_{r\theta} = E_{11}S_{r\theta}$.

State equations

$$
\begin{aligned}
\dot{x}_1 &= \left(x_2 - x_1 - x_4^2\right)/x_4, \\
\dot{x}_2 &= -\left[x_1\left(x_4\mathcal{S}'_{r\theta}u - \mathcal{S}_{rr}\right) - \mathcal{S}_{r\theta}x_4^2\right. \\
&\qquad \left. + x_2\left(x_4\mathcal{S}'_{\theta\theta}u + \mathcal{S}_{\theta\theta}\right)\right]/\left[x_4\mathcal{S}_{\theta\theta}\right], \\
\dot{x}_3 &= u, \\
\dot{x}_4 &= 1.
\end{aligned}
$$

where $\dot{x}_i = dx_i/d\tau,\ i = 1, 2, 3, 4$, $\mathcal{S}'_{\theta\theta} = d\mathcal{S}_{\theta\theta}/d\phi$, and $\mathcal{S}'_{r\theta} = d\mathcal{S}_{r\theta}/d\phi$.

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Flywheel Performance

- Energy density:

$$\alpha = \frac{\omega_f^2}{14400}\left(\frac{r_o^4 - r_i^4}{r_o^2 - r_i^2}\right) \text{ [Wh/kg]}$$

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- Energy density:

$$\alpha = \frac{\omega_f^2}{14400}\left(\frac{r_o^4 - r_i^4}{r_o^2 - r_i^2}\right) \text{ [Wh/kg]}$$

- Design problem: Find the radial-ply fiber orientation that will maximize ω_f.

Failure theories: When does the disk fail?

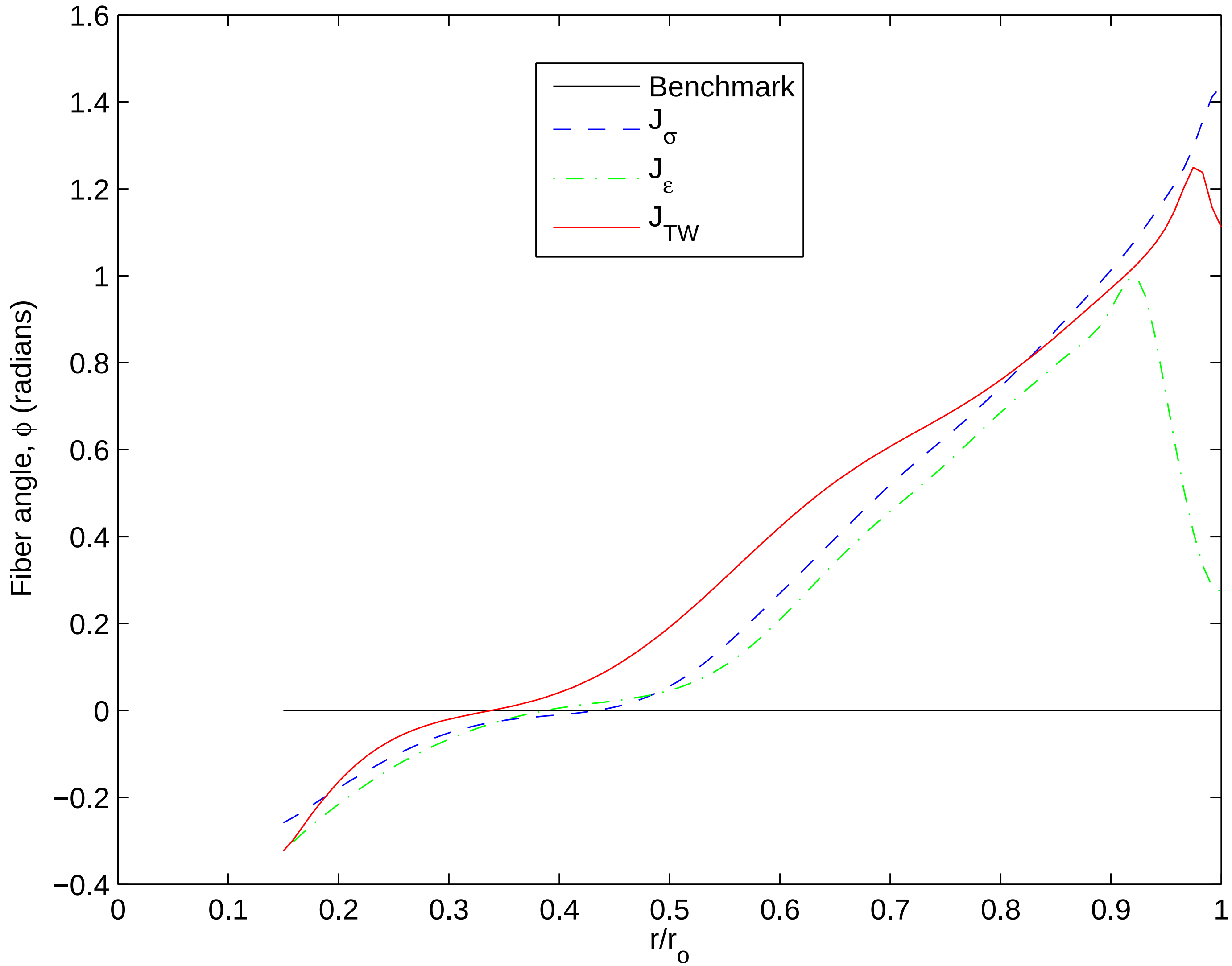
- Maximum stress failure criterion
- Maximum strain failure criterion
- Tsai-Wu failure criterion

Optimization results

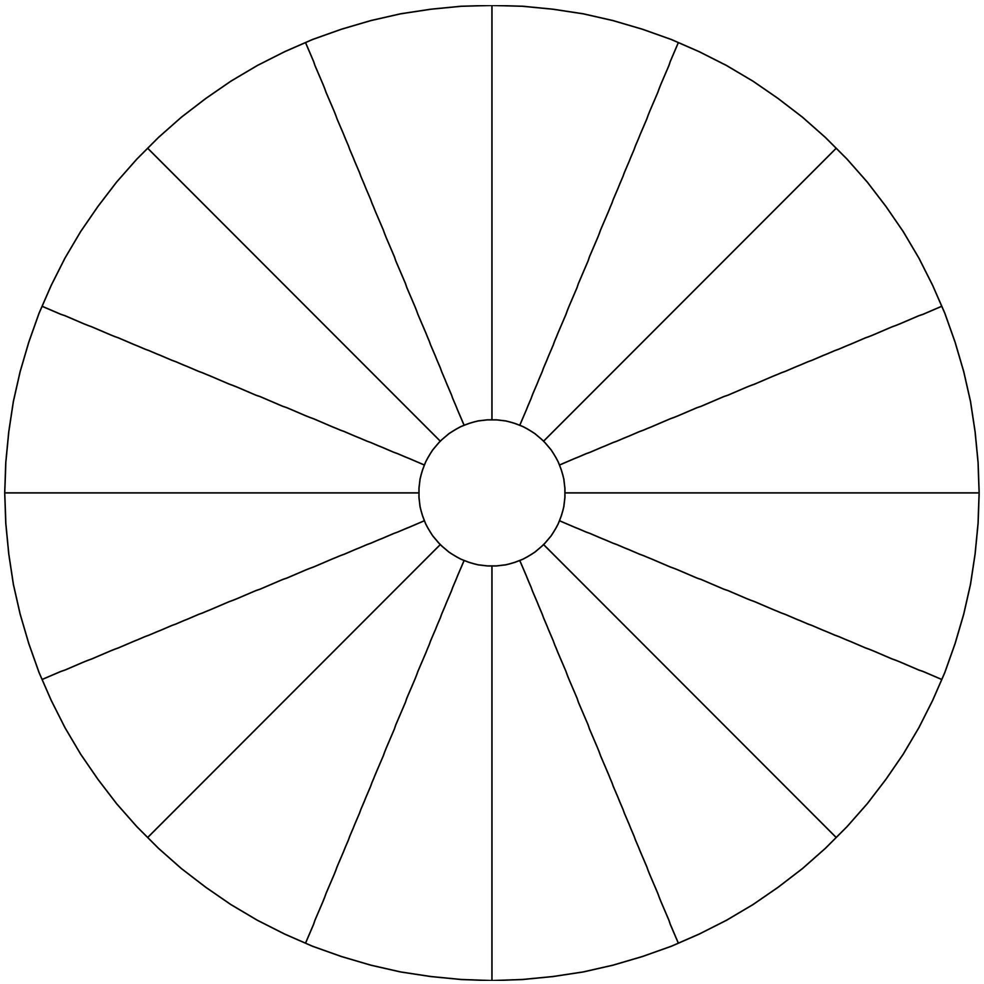
Compare 4 designs:

- Benchmark design: $\phi(\tau) = 0$.
- J_σ, maximum stress failure criterion.
- J_ϵ, maximum strain failure criterion.
- J_{TW}, Tsai-Wu failure criterion.

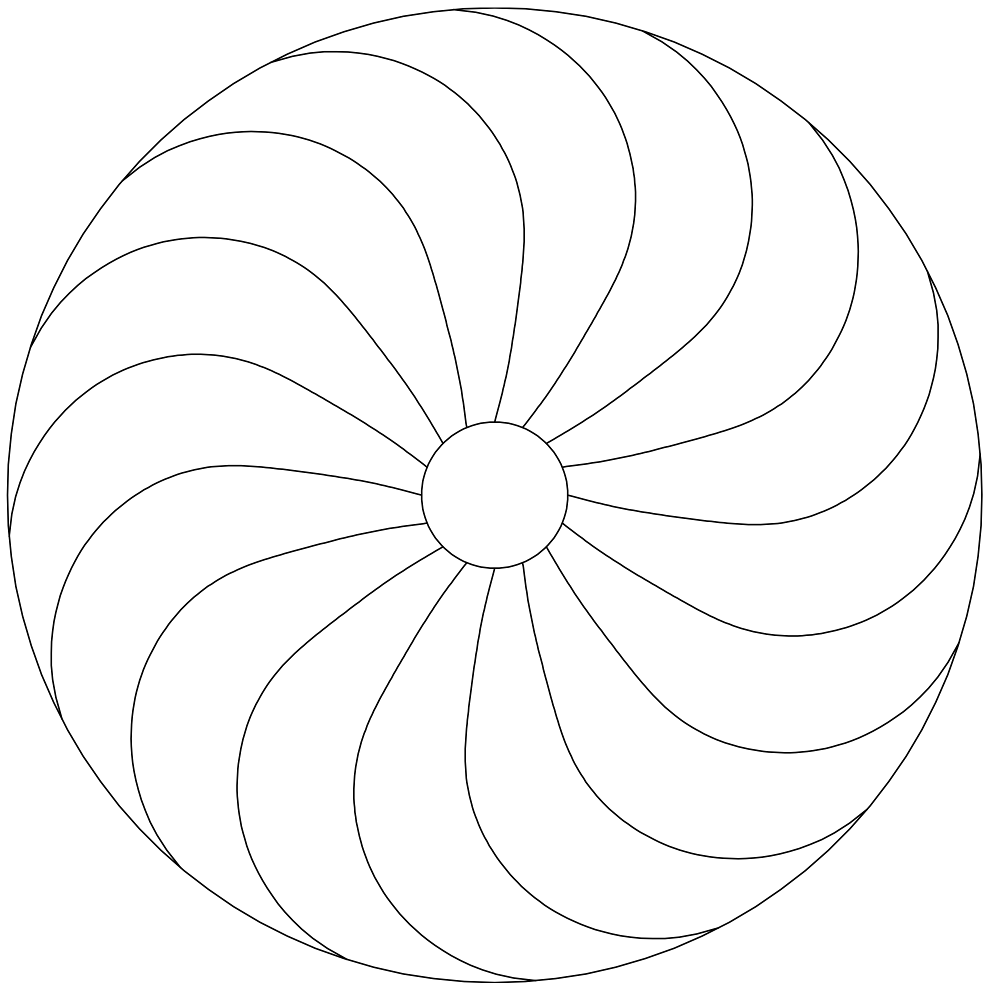
Fiber orientation



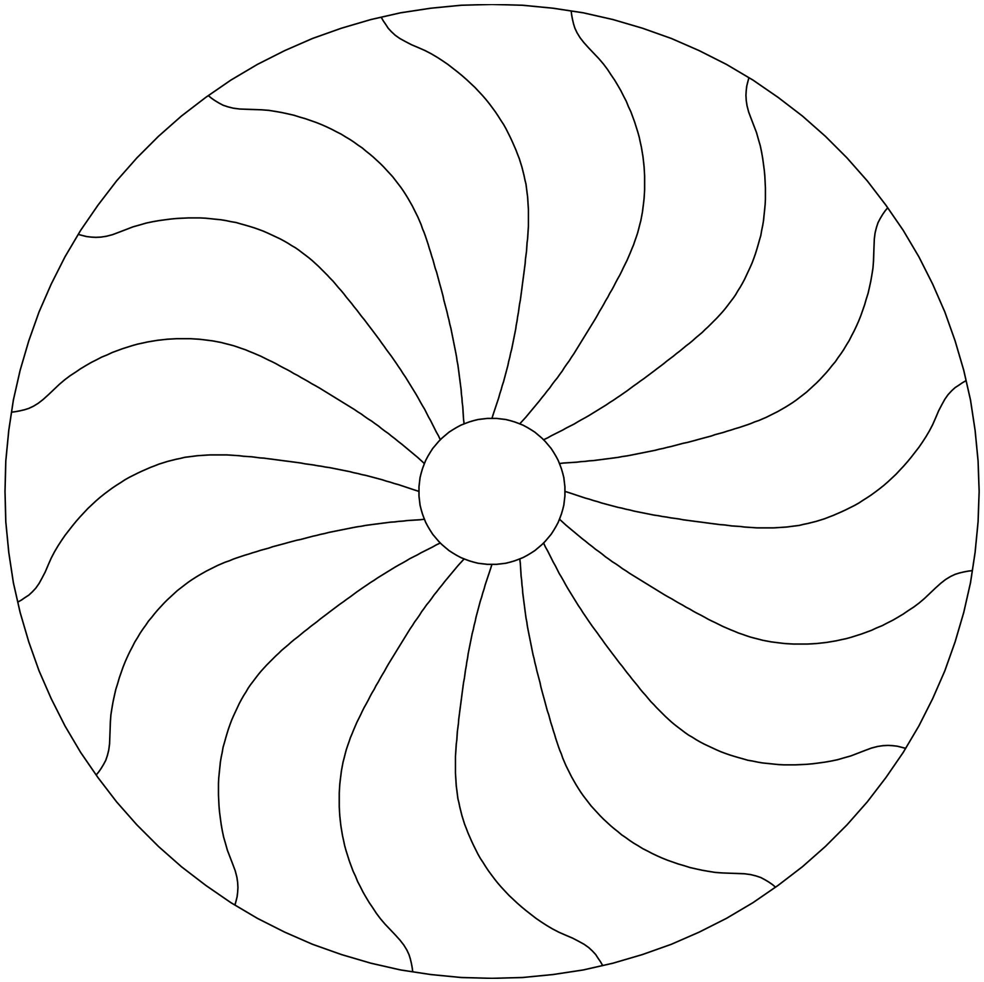
Benchmark design



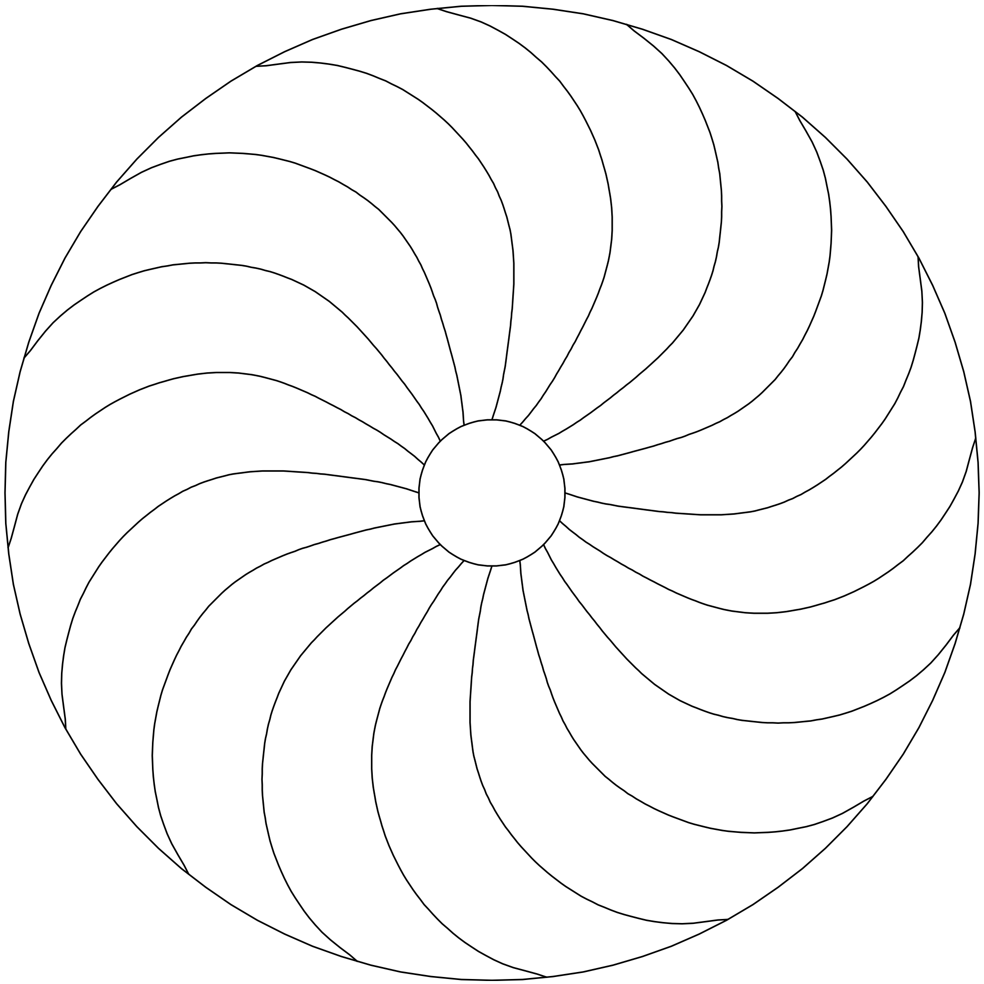
Maximum stress design



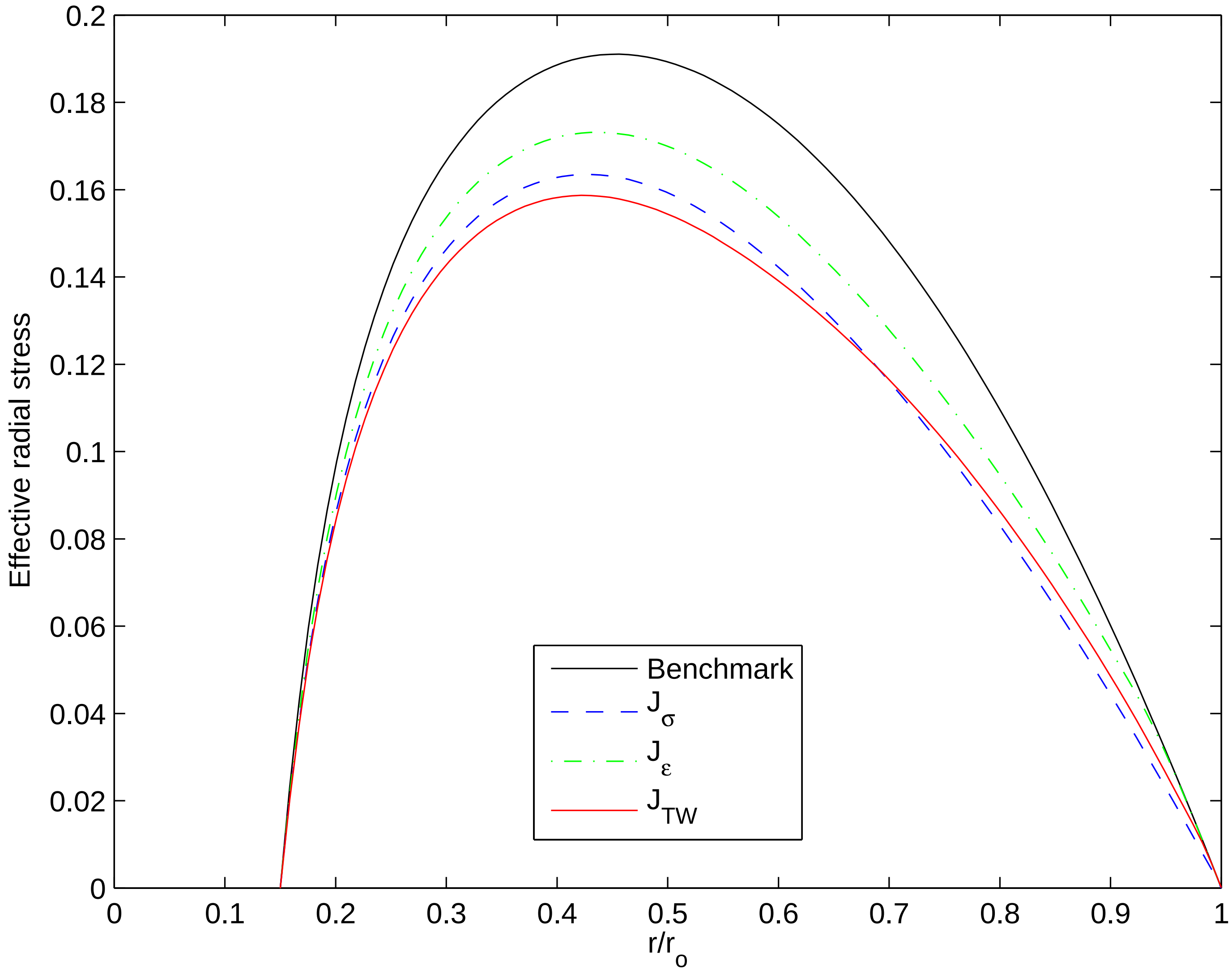
Maximum strain design



Tsai−Wu design



Effective radial stress



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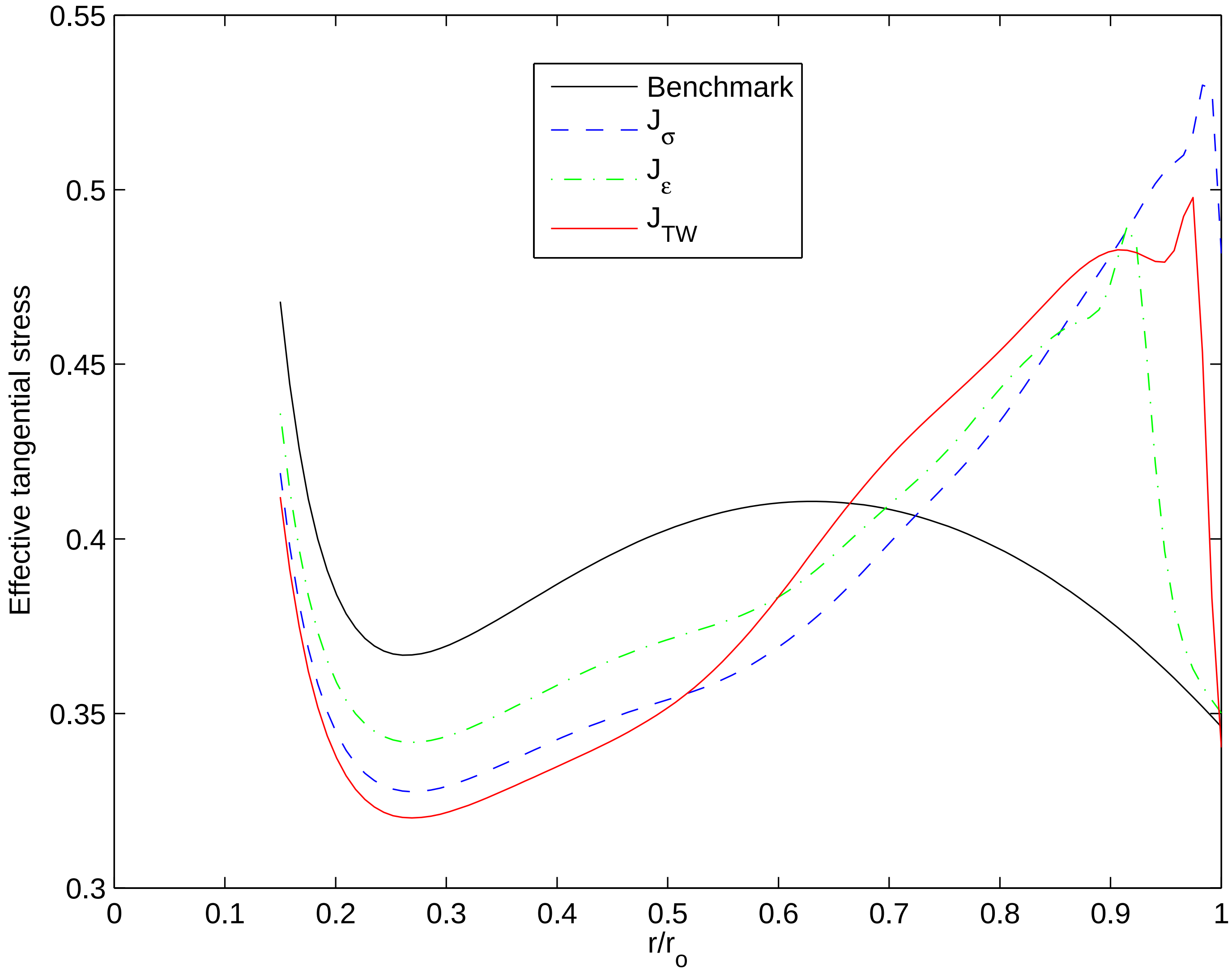
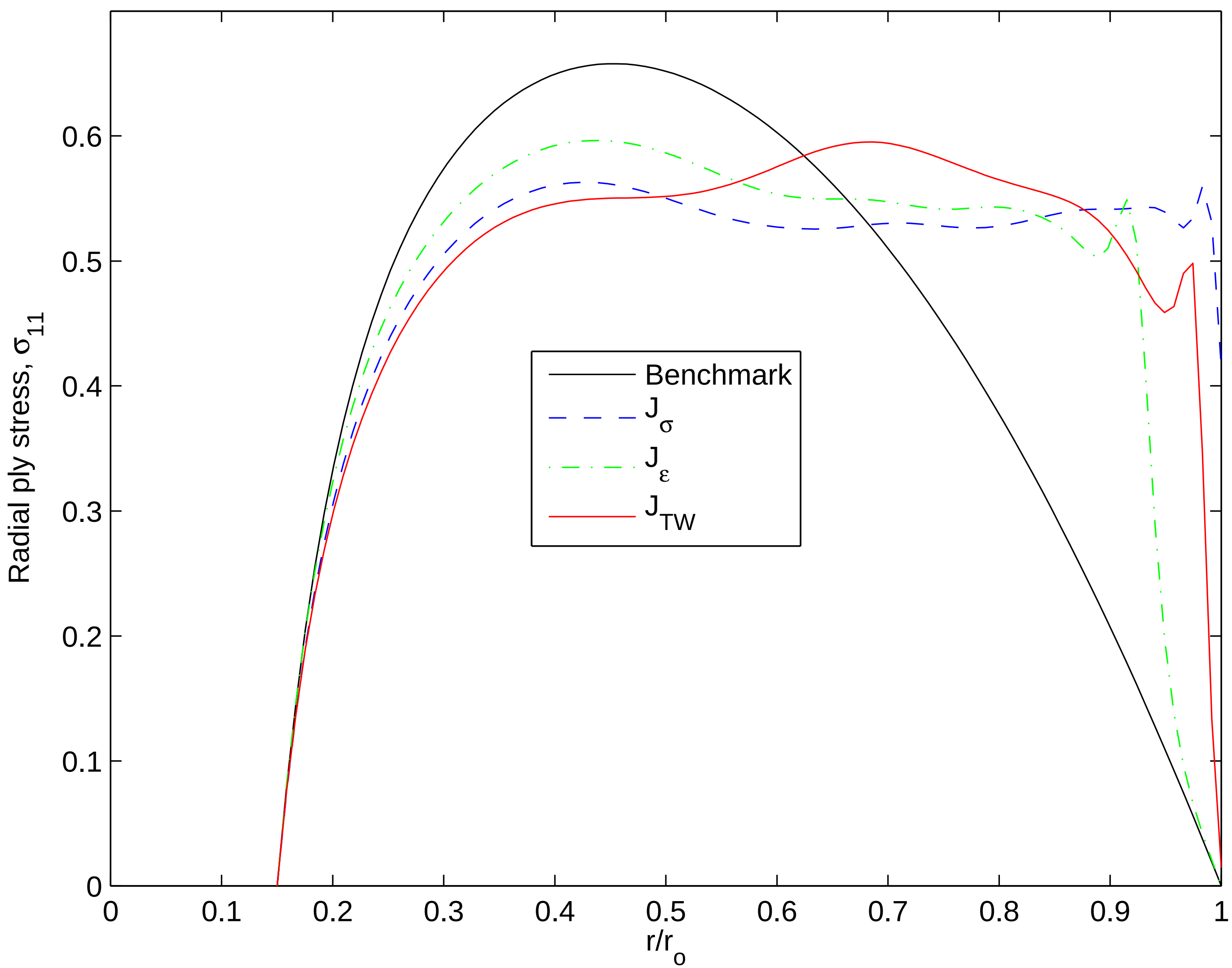
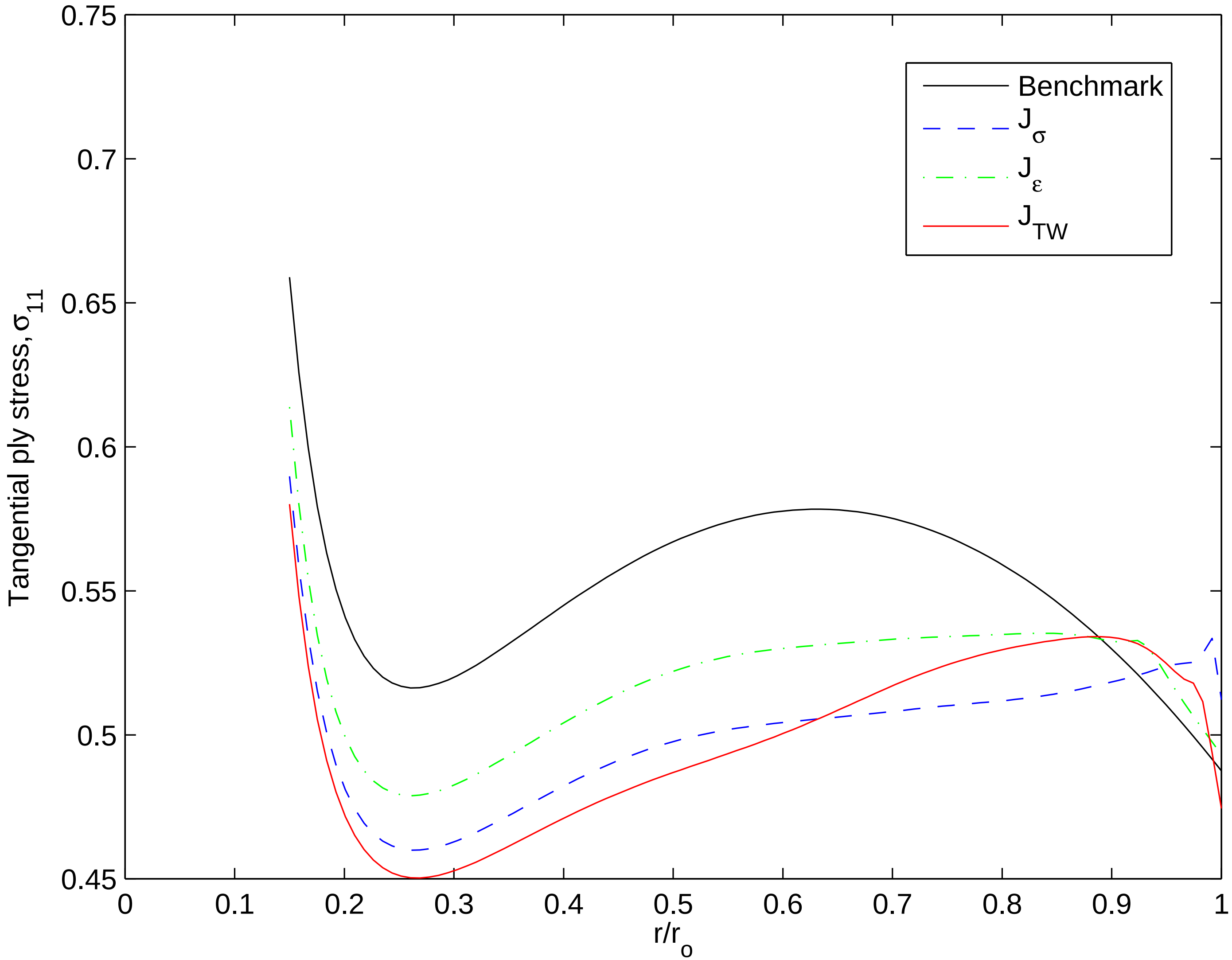
Effective tangential stress



Radial ply stress σ_{11}



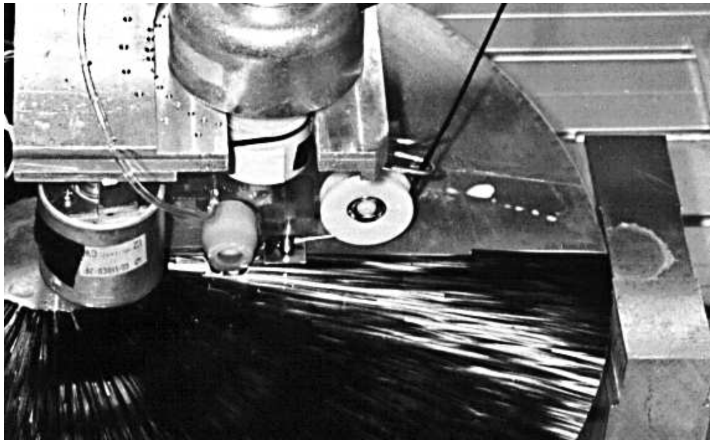
Tangential ply stress σ_{11}

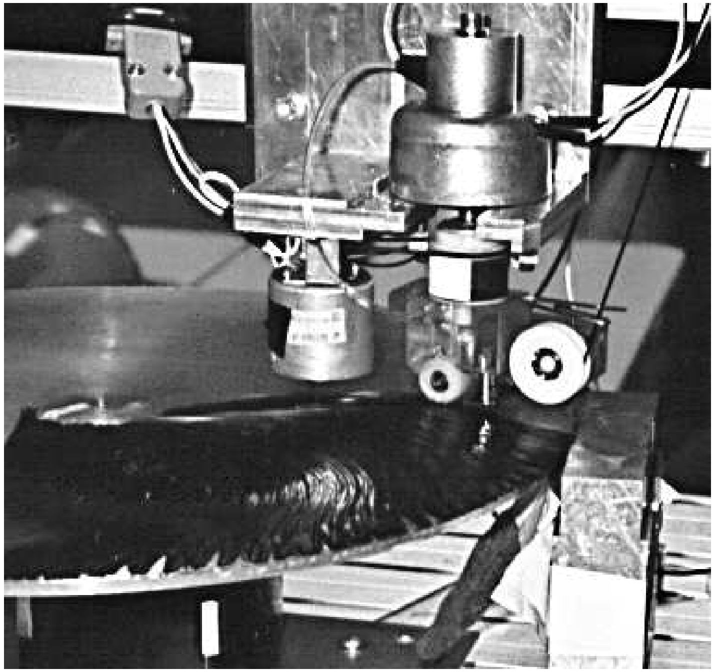


Energy density

Disk design	Maximum stress	Maximum strain	Tsai-Wu
Benchmark	150	150	142
J_σ	168	168	106
J_ϵ	161	161	157
J_{TW}	166	166	156

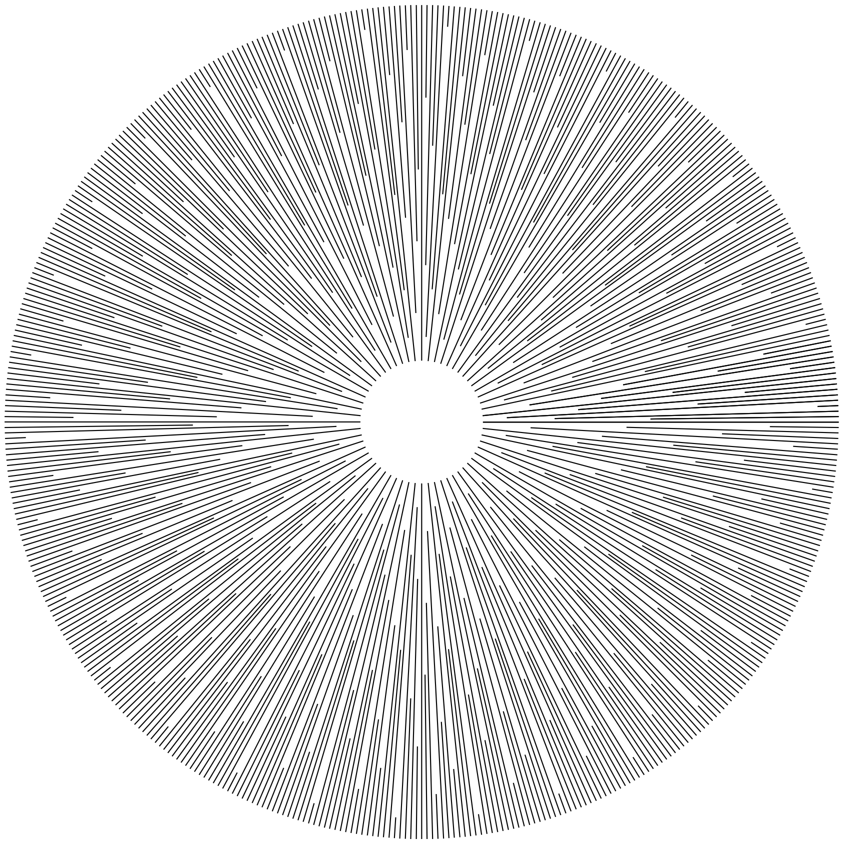
Flywheel construction





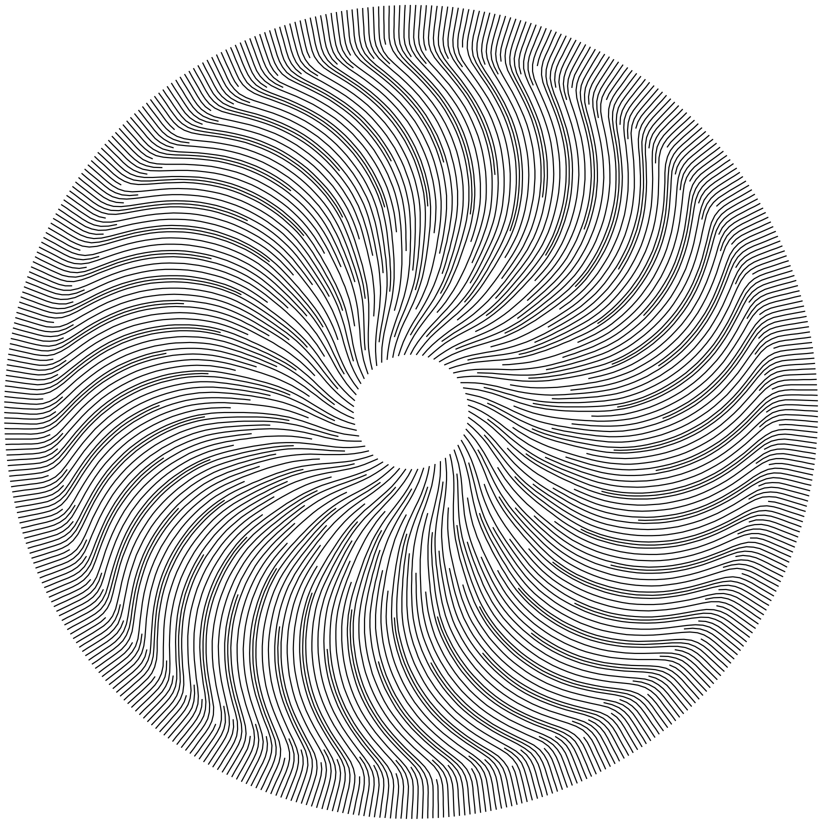
Fiber layout

Benchmark design

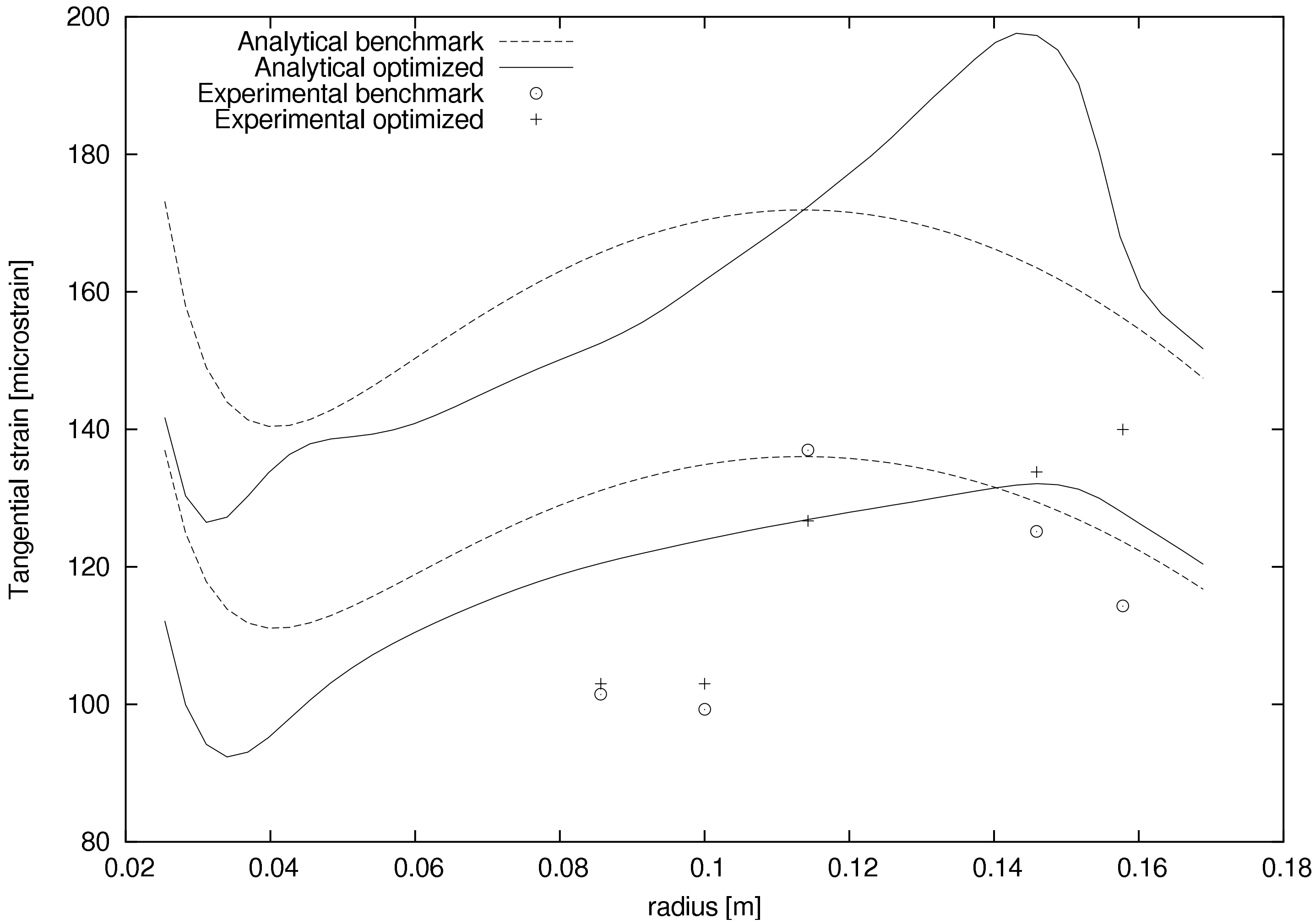


Fiber layout

Optimized design



Tangential strain



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Conclusions

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