Development of Reliability-Based Damage Tolerant Structural Design Methodology

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FAA Sponsored Project Information

- **Principal Investigator:**
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- **Pre-Doctoral Research Assistant:** Chi Ho “Eric” Cheung, UW
- **Undergraduate Research Assistant:** Phillip Gray, UW
- **FAA Technical Monitor:** Curtis Davies
- **Other FAA Personnel:** Larry Ilcewicz
- **Industry Participants:** Gerald Mabson, Eric Cregger, Marc Piehl, Cliff Chen, Lyle Deobald, Alan Miller (All from Boeing)
- **Industry Sponsors:** Boeing

Accomplishments

Work Accomplished: Phase 1

- Developed the methodology to determine the reliability and maintenance planning of damage tolerant structures.
- Developed a user-friendly software (RELACS) for calculating POF and inspection intervals.
- Developed software interface (VSTM) with Nastran to facilitate stochastic FEA.
- Implemented stochastic FEA to obtain initial/damaged residual strength variance.

Current Research

- Develop analytical methods to analyze disbond and delamination arrest mechanisms in bonded structures under mixed mode loading.
- To apply probabilistic methods to assess reliability of bonded structures with fasteners.

Analysis of Disbond/Delamination Arrest Mechanisms

- **Objectives**

  - To understand the effectiveness of delamination/disbond arrest mechanisms

  - To develop analysis tools for design and optimization

- **Tasks**

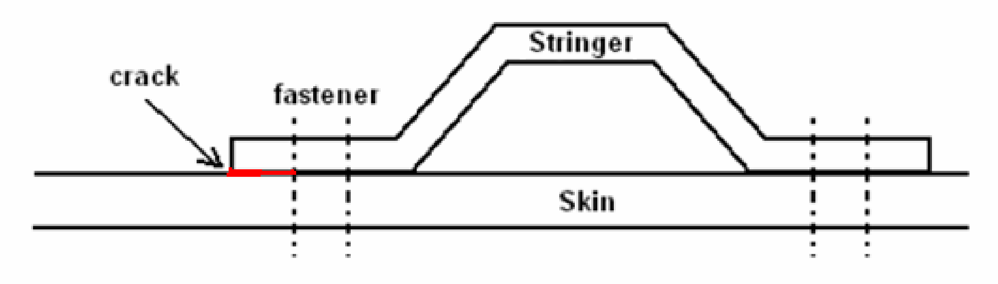
  1). Establish FE models in ABAQUS

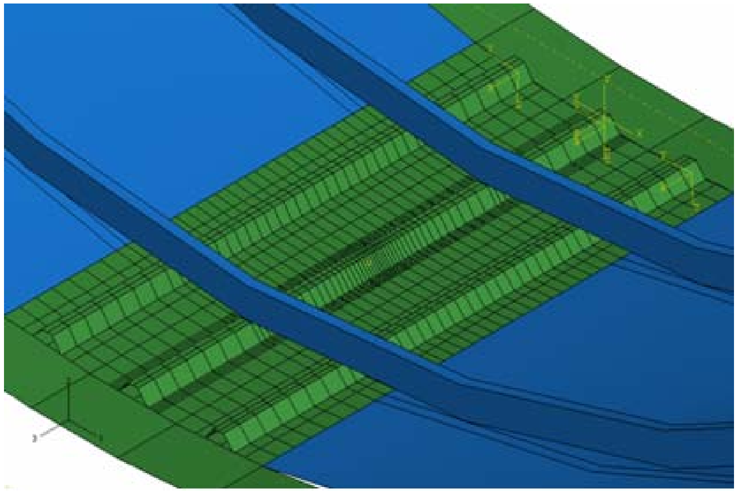
  2). Develop 1-D(beam) and 2D (plate) analytical capabilities

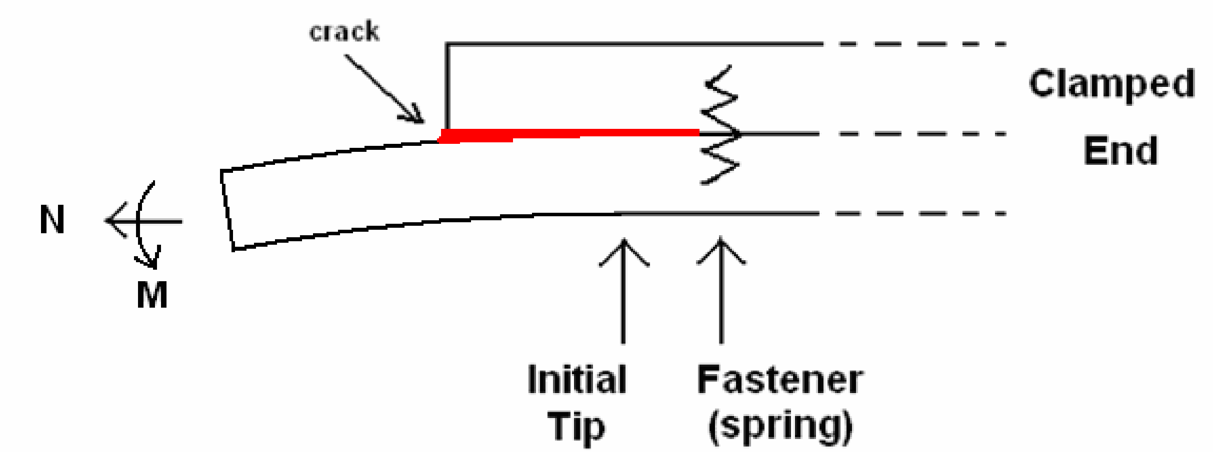
  3). Implement reliability analysis capability

  4). Conduct sensitivity studies on fastener effectiveness and stacking sequence effects

Bonded Skin/Stiffener with Fasteners







Model Description

- 16-ply CFRP (t = 0.0075” x 16 = 0.12”)
- Lay-ups
  - Percentage of 0-deg: 25% / 37.5% / 50% / 62.5%
- Fastener
  - Ti-Al6-V4 (E = 16.5x10^6psi)
  - d = 0.25 in
- Fastener Flexibility (H. Huth, 1986)

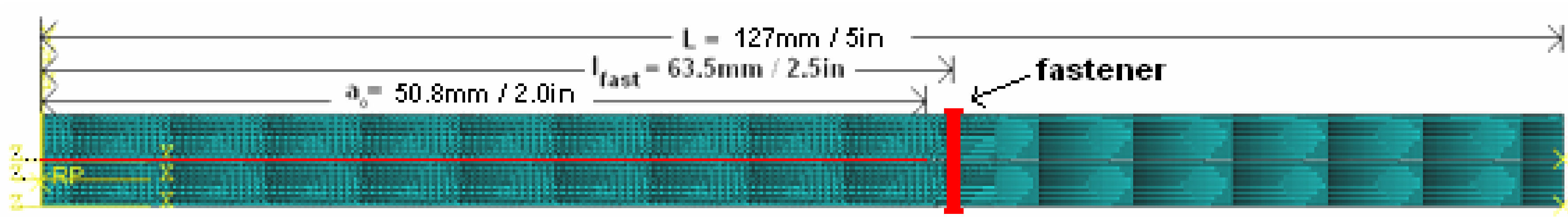
$$C = \left(\frac{t_1 + t_2}{2d}\right)^a \frac{b}{n}\left(\frac{1}{t_1 E_1} + \frac{1}{n t_2 E_2} + \frac{1}{n t_1 E_3} + \frac{1}{2 n t_2 E_3}\right)$$

Model Description

- B-K law for mixed-mode VCCT criteria

$$G_{equivC} = G_{IC} + \left(G_{IIC} - G_{IC}\right)\left(\frac{G_{II}}{G_I + G_{II}}\right)^{\eta}$$

- Fastener failure not considered
- Fastener pull-through not considered



Laminate Configuration (16 plies)

0-ply	Lay-up	E_x	C (in/lb) (fastener compliance)
25.0%	$(45/0/-45/90/45/0/-45/90)_s$	7.42×10^{6}	7.73×10^{-6}
37.5%	$(45/0/-45/0/45/0/-45/90)_s$	9.29×10^{6}	6.57×10^{-6}
50.0%	$(45/0_2/-45/0_2/90_2)_s$	1.10×10^{7}	5.85×10^{-6}
62.5%	$(45/0_3/-45/0_2/90)_s$	1.30×10^{7}	5.25×10^{-6}

$$C=\left(\frac{t_1+t_2}{2d}\right)^a \frac{b}{n}\left(\frac{1}{t_1E_1}+\frac{1}{nt_2E_2}+\frac{1}{nt_1E_3}+\frac{1}{2nt_2E_3}\right)$$

$$a=2/3,\ b=4.2,\ n=1$$

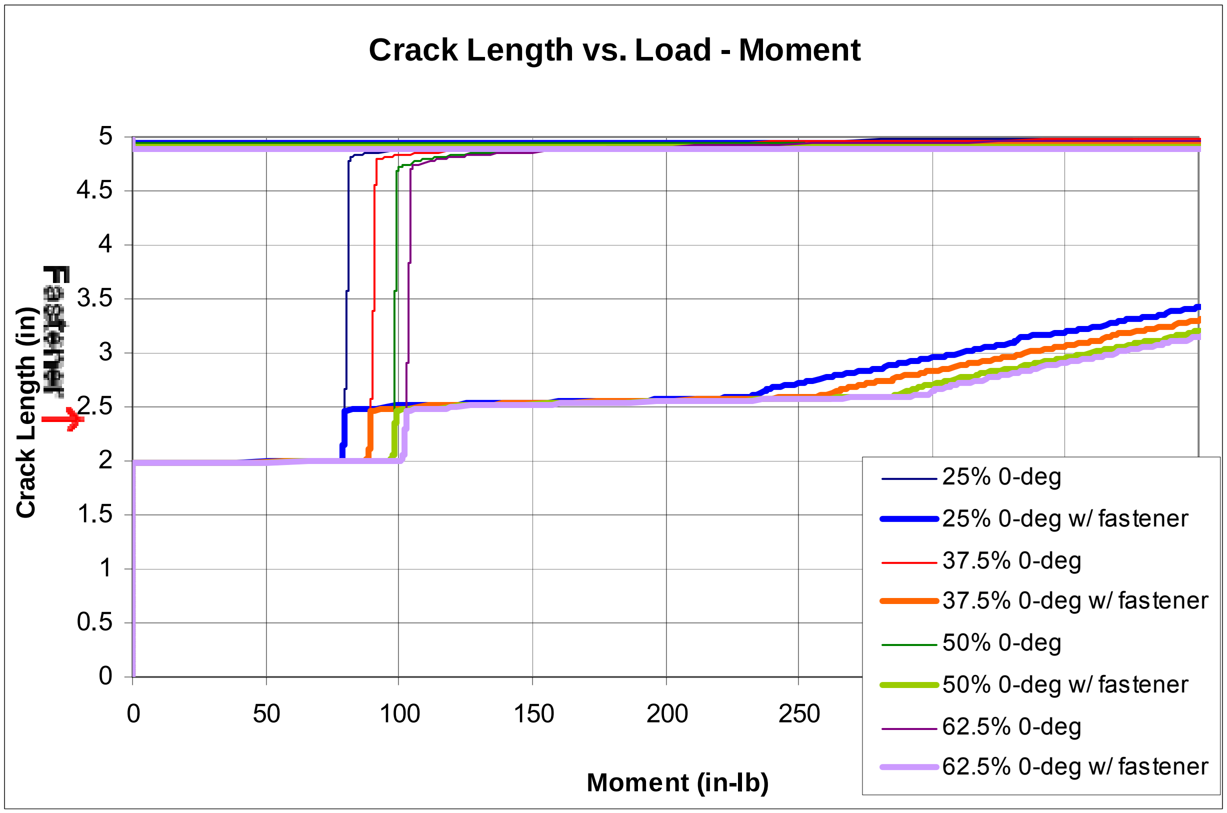
$$k_{clamp}=\frac{AE}{(t_1+t_2)}=3.37\times10^6$$

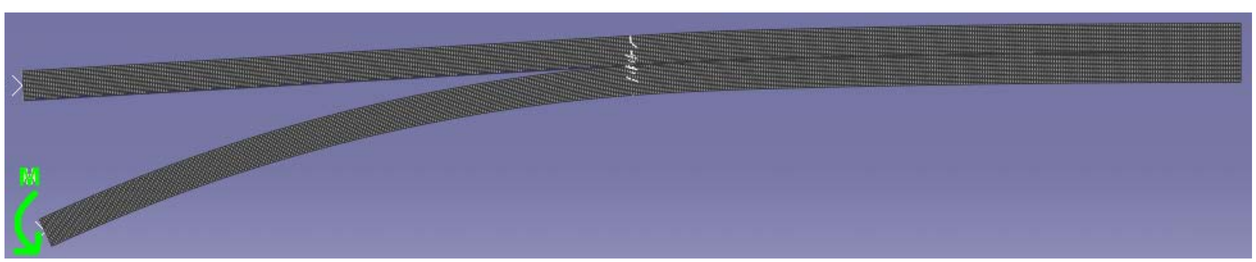
Material Properties (AS4/3501-6)

- E_1=127.5GPa
- E_2=11.3GPa
- G_{12}=6.0GPa
- v=0.3
- X_t=2282MPa
- X_c=1440MPa
- Y_t=57MPa
- Y_c=228MPa
- S_{xy}=71MPa
- G_{IC}=0.2627N/mm
- G_{IIC}=1.226N/mm
- η=1.75

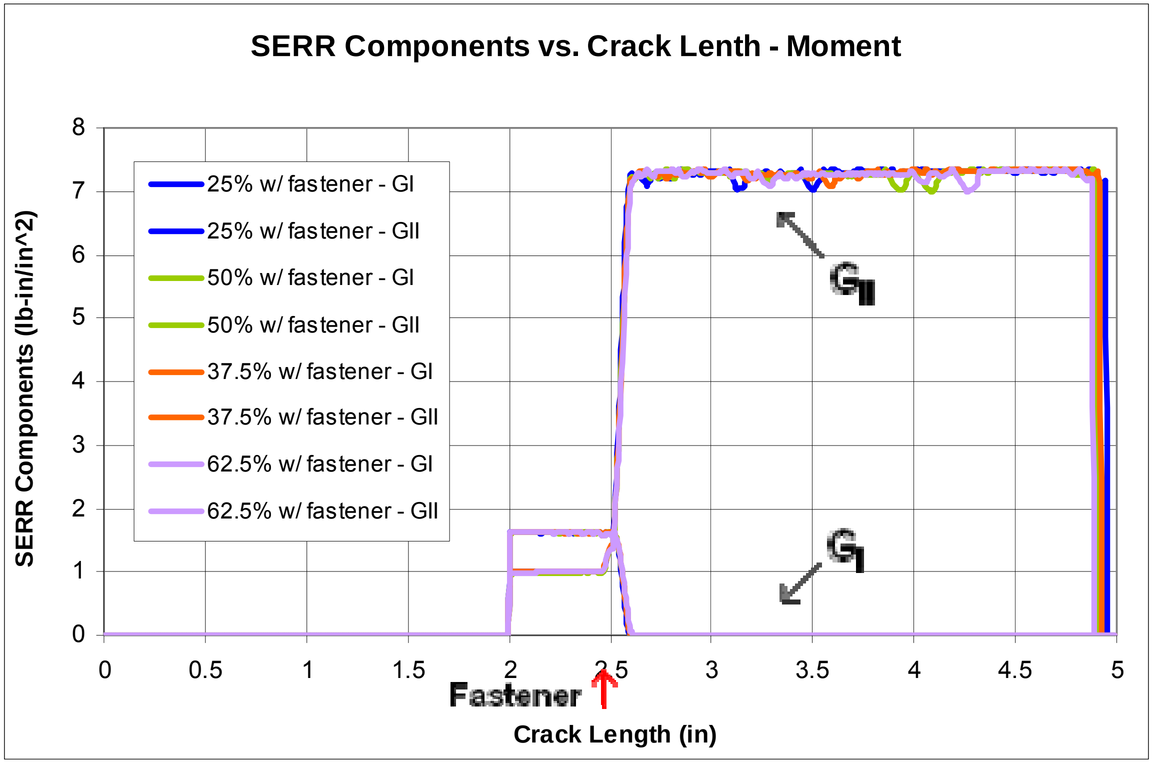
- E_1=18.5Msi
- E_2=1.64Msi
- G_{12}=0.871Msi
- v=0.3
- X_t=331ksi
- X_c=208.9ksi
- Y_t=8.3ksi
- Y_c=33.1ksi
- S_{xy}=10.3ksi
- G_{IC}=1.5lb/in
- G_{IIC}=7.0lb/in
- η=1.75

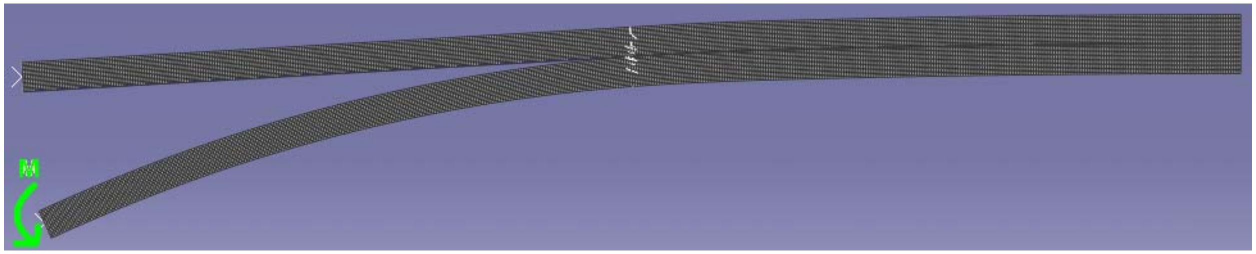
Results: Applied Moment M Only



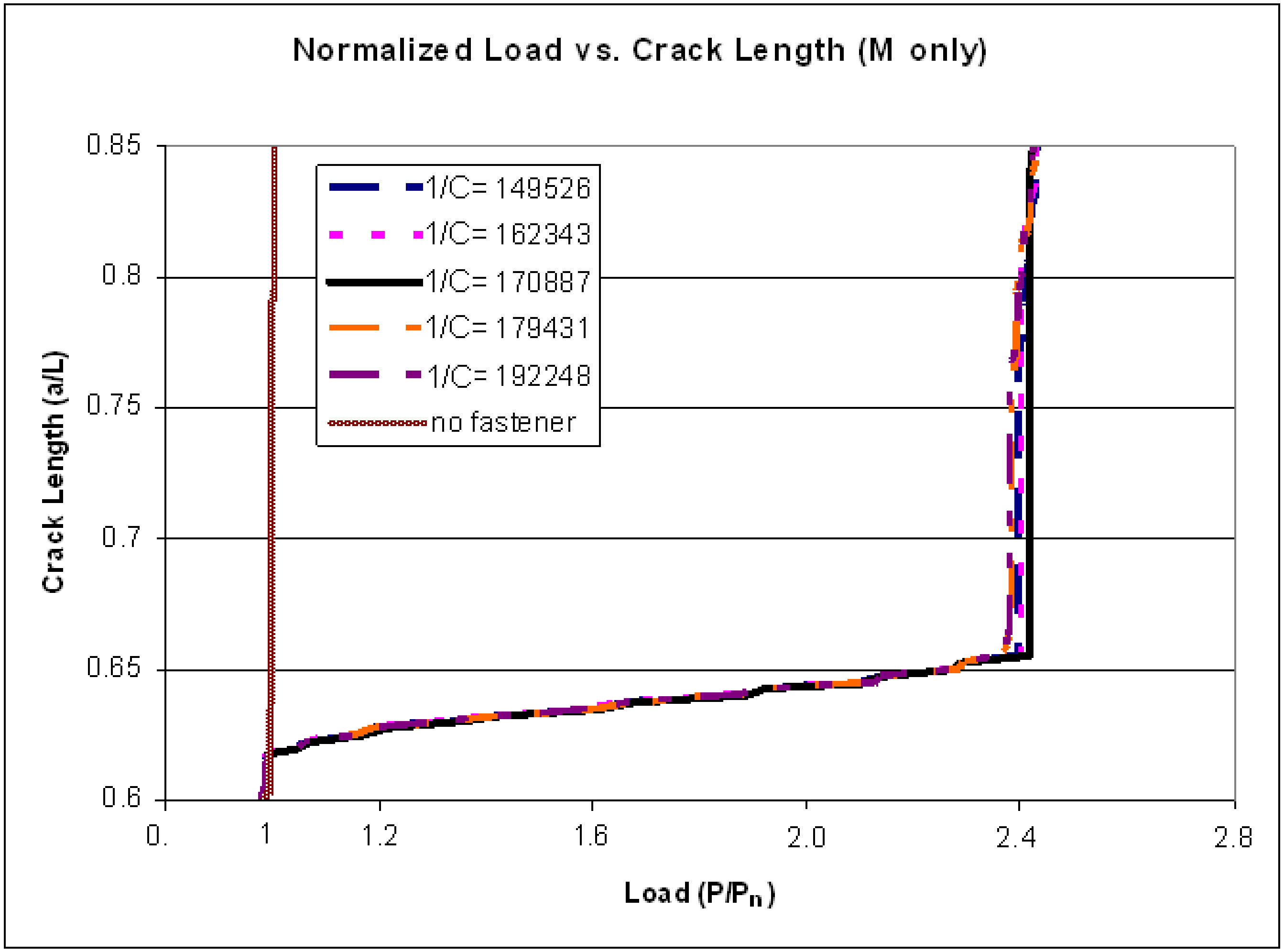


Mode Decomposition with Fastener: Applied Moment M Only

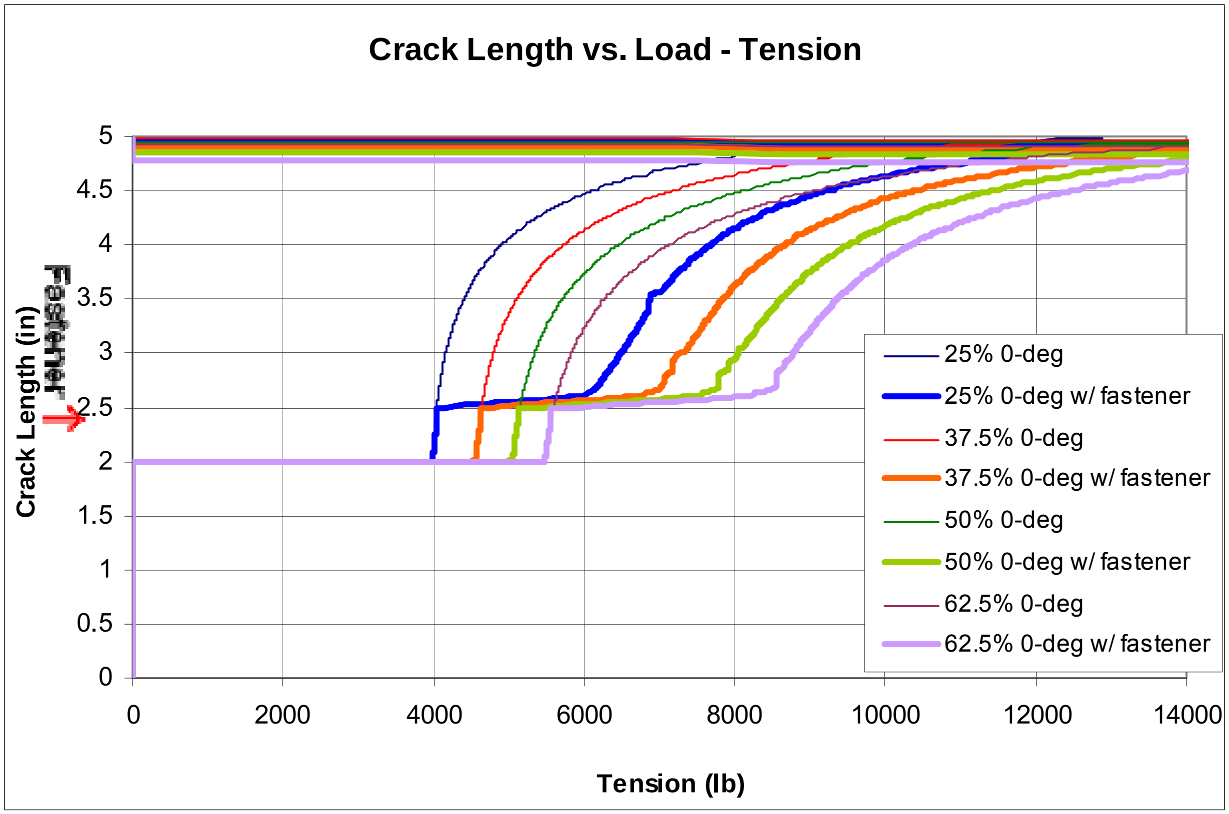


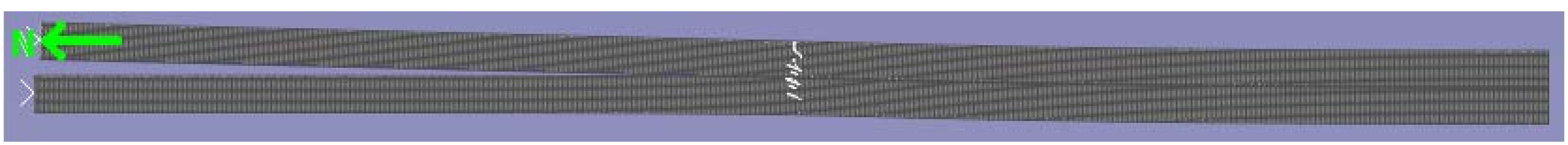


Sensitivity with Respect to Fastener Flexibility: Applied Moment M Only

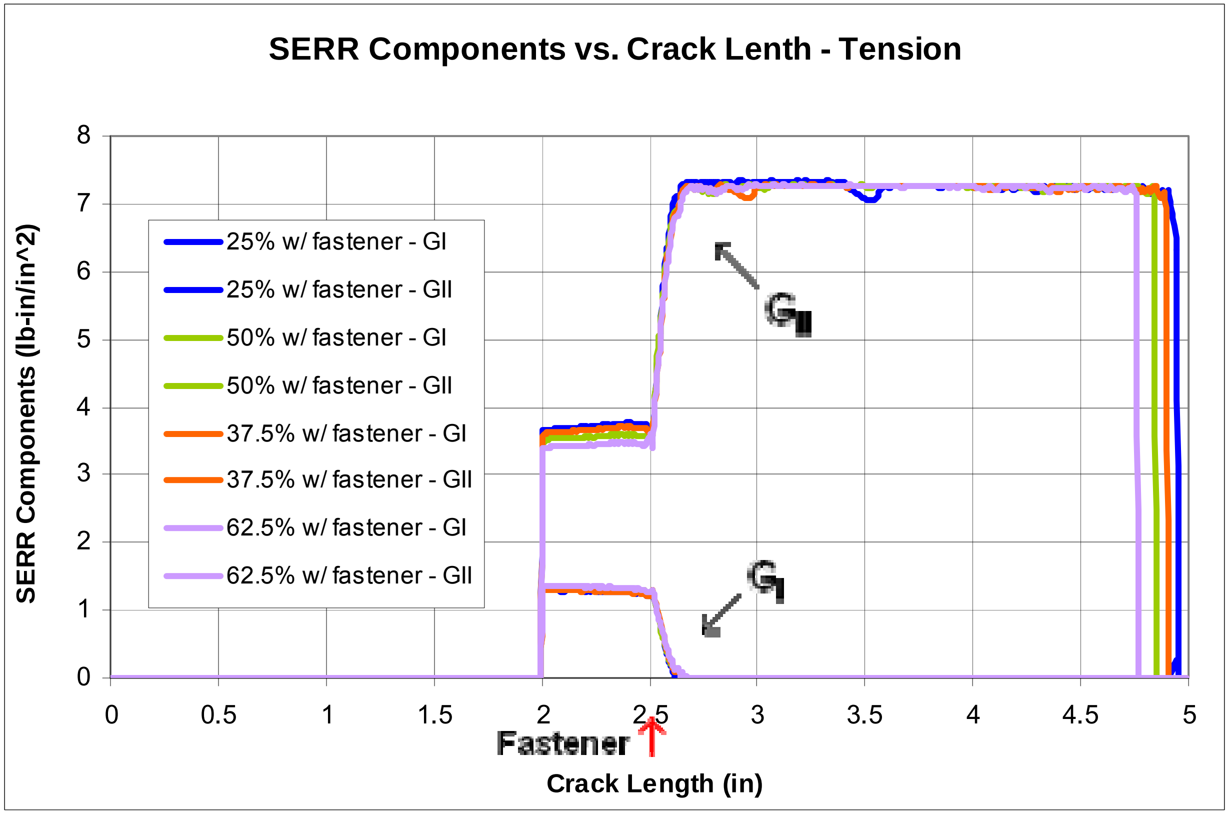


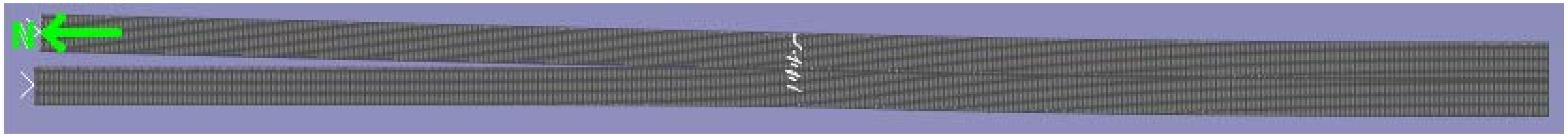
Results: Applied Tension N Only



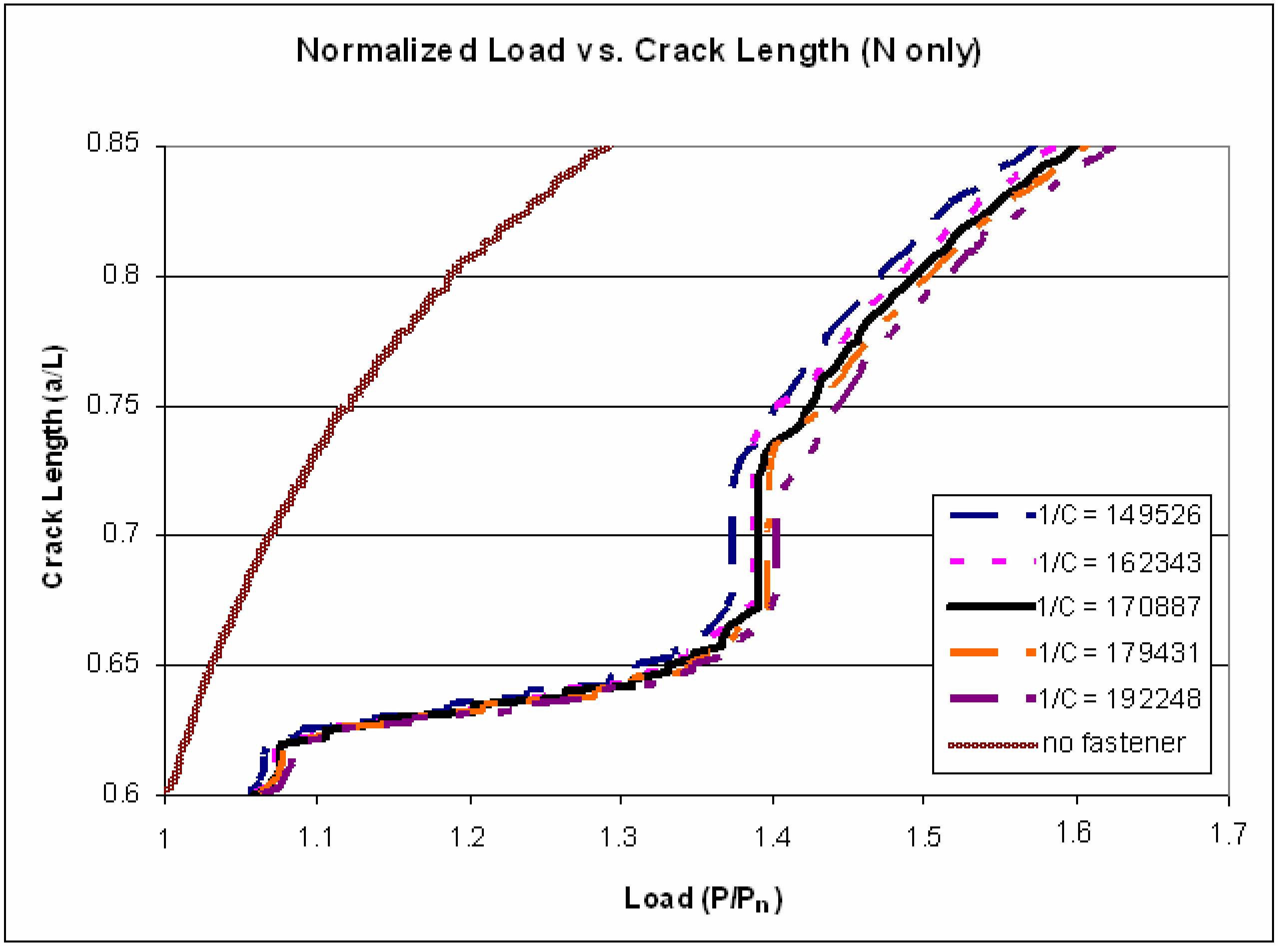


Mode Decomposition: Applied Tension N Only

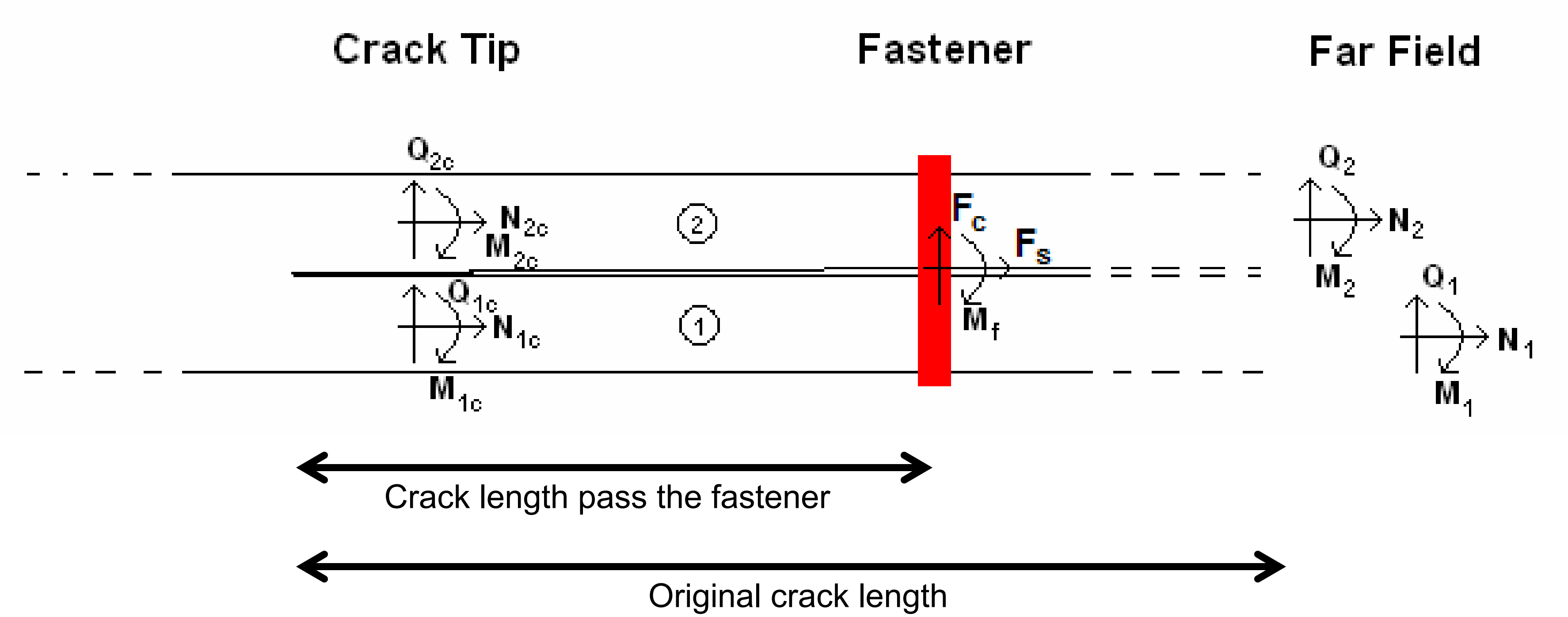




Sensitivity with Respect to Fastener Flexibility: Applied Moment M Only

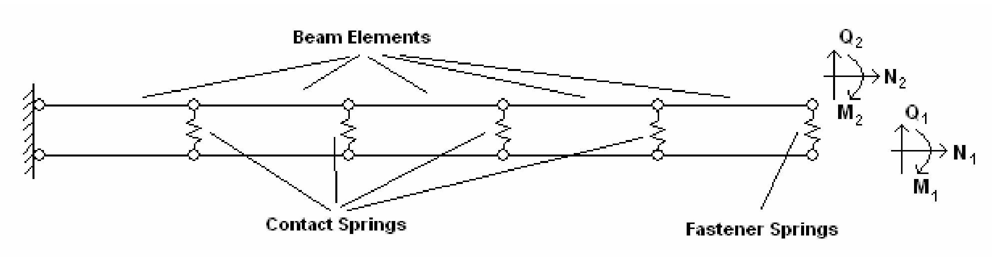


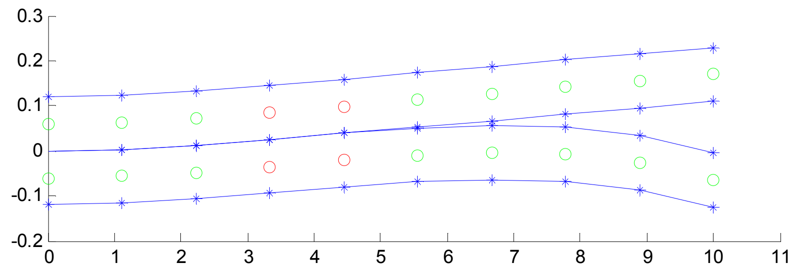
Analytical Model



1. FEM Approach

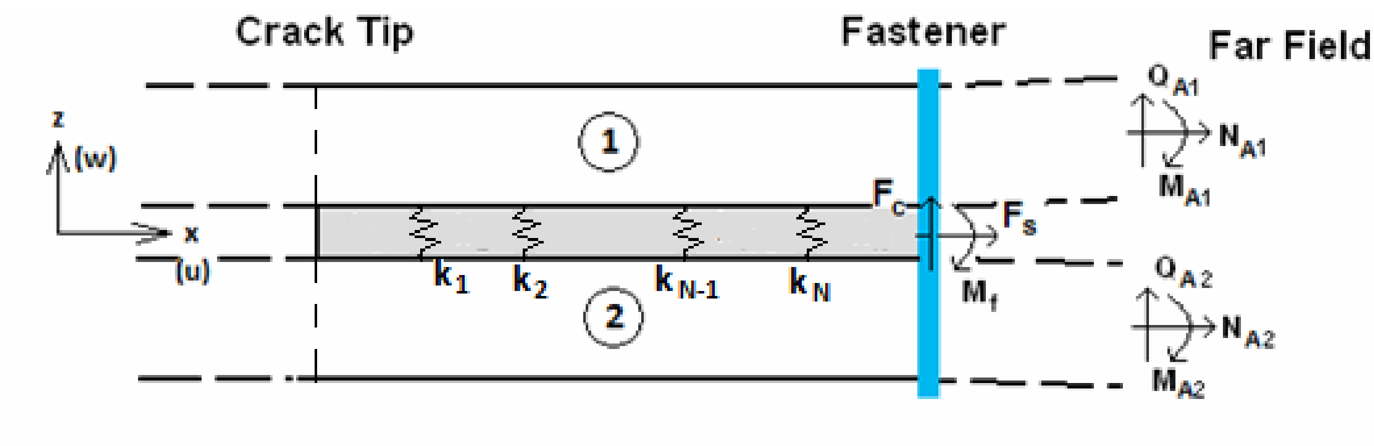
- Traditional FEM framework
- Equivalent orthotropic material properties (Ex, D, Gxz)
- Shear-deformable “frame” elements with reduced integration
- Axial and transverse force/moment are considered simultaneously
- Contact springs are placed along the length of the beam to resolve penetration
- Fasteners can be modeled with springs in the x, y and θ direction
- Equilibrium must be solved iteratively
- SERR mode decomposition by VCCT





2. Analytical Approach

- Uses Rayleigh-Ritz method and the energy principle
- Two beams, fastener (three springs), and elastic-foundation layer between the beams
- Elastic layer is composed of N individual springs where k is very large in compression and zero in tension, for contact and separation
- Solve system for the state of minimum potential energy iteratively
- SERR mode decomposition by VCCT



Analytical Approach

$$\delta \Pi = 0; \quad where\ \Pi = U_{total} - W_{total}$$

Beam Energy Terms

$$U_b = \frac{1}{2} EI \int_0^L \left(\frac{d^2 w}{dx^2}\right)^2 dx$$

$$U_s = 1.2 \frac{EI^2}{A} (1 + v) \int_0^L \left(\frac{d^3 w}{dx^3}\right)^2 dx$$

$$U_a = \frac{1}{2} AE \int_0^L \left(\frac{du}{dx}\right)^2 dx$$

$$U_{ba} = \frac{1}{2} N \int_0^L \left(\frac{dw}{dx}\right)^2 dx$$

Elastic Layer Energy

$$U_{EL} = \sum_{n=1}^{N} \frac{1}{2} k_n \left(w_2 - w_1\right)^2 \Big|_{x=L\left(\frac{n}{N}\right)}$$

Fastener/Spring Energy

$$U_{kc} = \frac{1}{2} k_c (w_2 - w_1)^2 \big|_{x=L}$$

$$U_{ks} = \frac{1}{2} k_s (u_2 - u_1)^2 \big|_{x=L}$$

$$U_{ks} = \frac{1}{2} k_r \left(\frac{dw_2}{dx} - \frac{dw_1}{dx}\right)^2 \Big|_{x=L}$$

Shape Functions

- Shape functions must satisfy geometric boundary conditions.
  - Shape functions considered for transverse displacement.

$$w_1 = \sum_{i=1}^{I} \alpha_i x^{m+1} \; ; \; i = 1,2,3 \ldots I$$

$$w_2 = \sum_{j=1}^{J} \beta_j x^{n+1} \; ; \; j = 1,2,3 \ldots J$$

$$w_1 = \sum_{i=1}^{I} \alpha_i \left[1 - \cos\left(\frac{i\,\pi}{2\,L} x\right)\right] \; ; \; i = 1,3,5 \ldots I$$

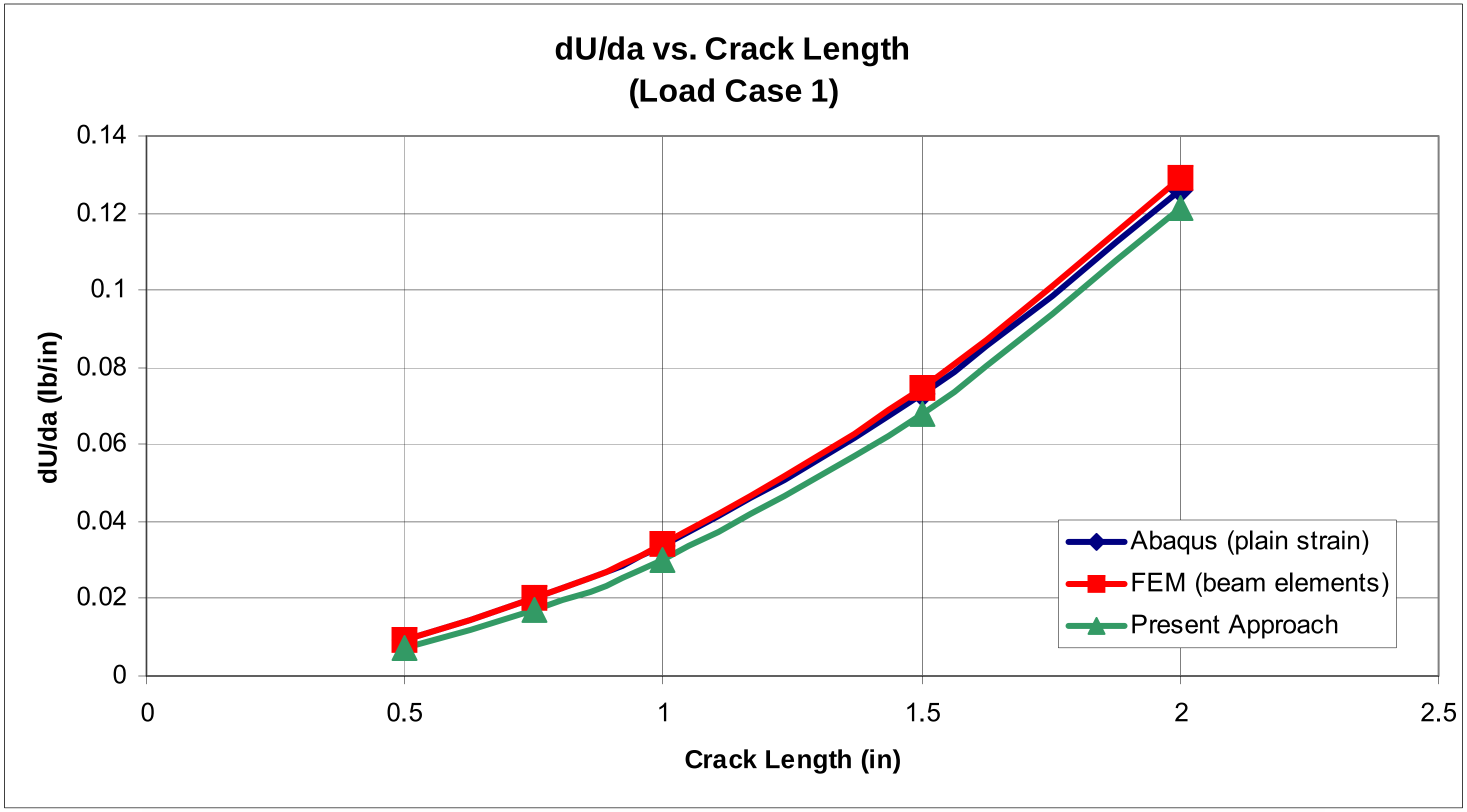
$$w_2 = \sum_{j=1}^{J} \beta_j \left[1 - \cos\left(\frac{j\,\pi}{2\,L} x\right)\right] \; ; \; j = 1,3,5 \ldots J$$

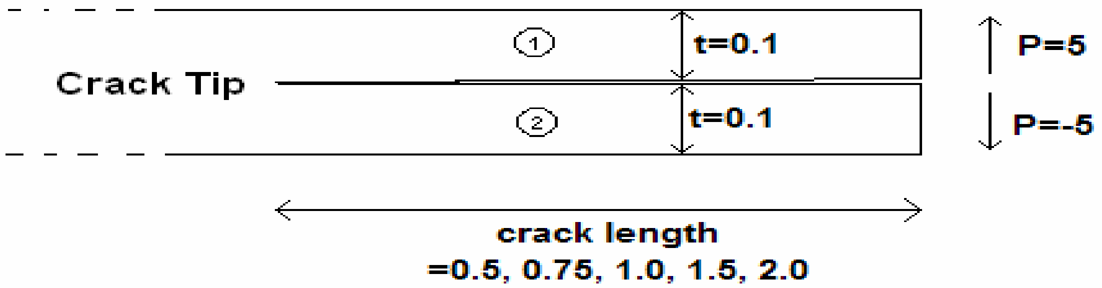
  - Shape functions considered for axial displacement.

$$u_1 = x\left(1 + \frac{N_1}{A_1 E_1}\right) \qquad u_2 = x\left(1 + \frac{N_2}{A_2 E_2}\right)$$

Comparison of Results

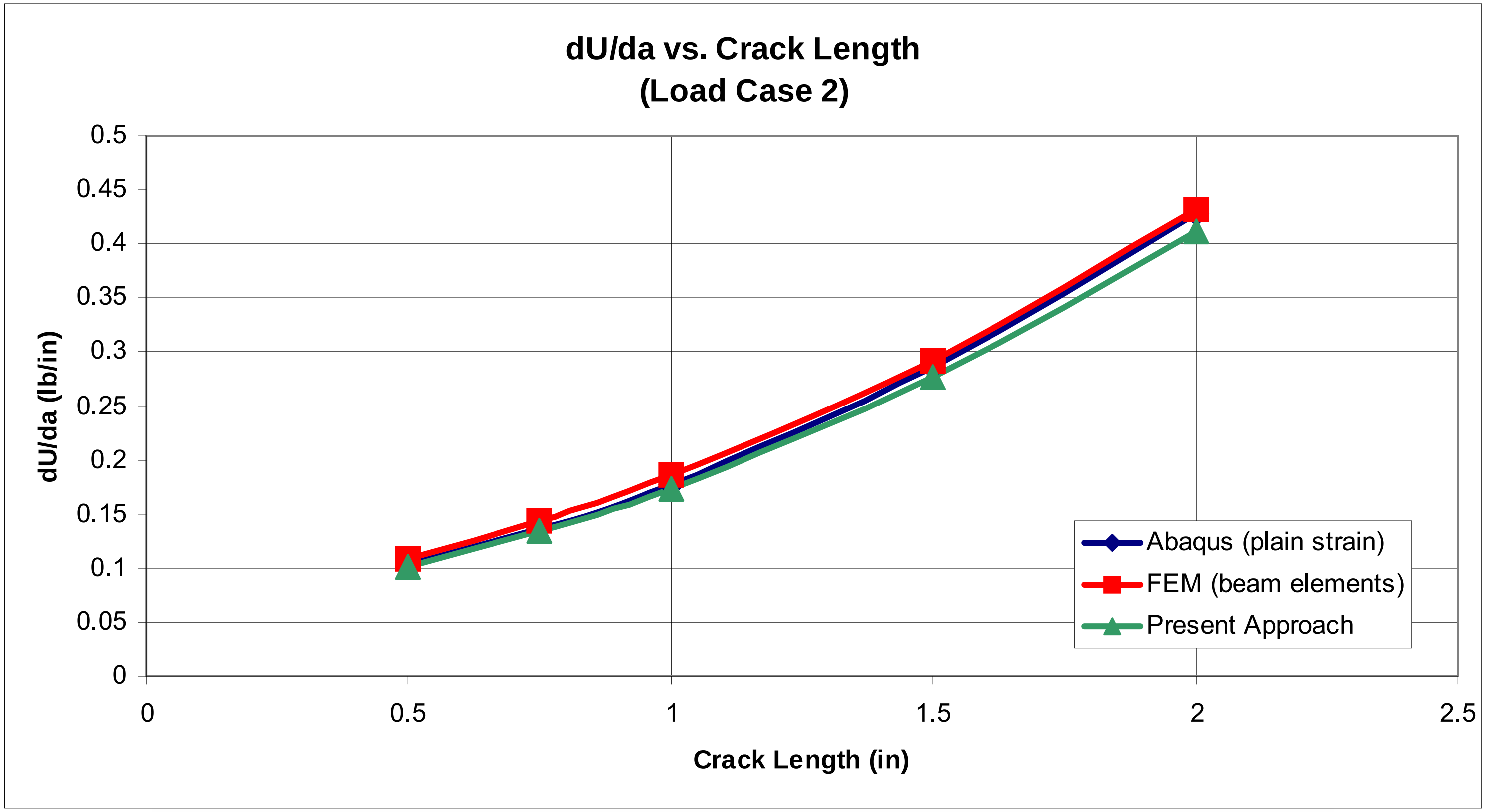
(Load case 1, $\Delta a/a$ = 1%)

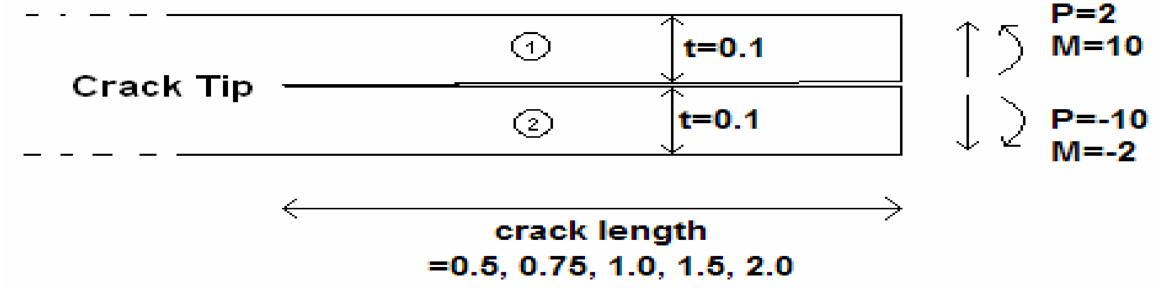




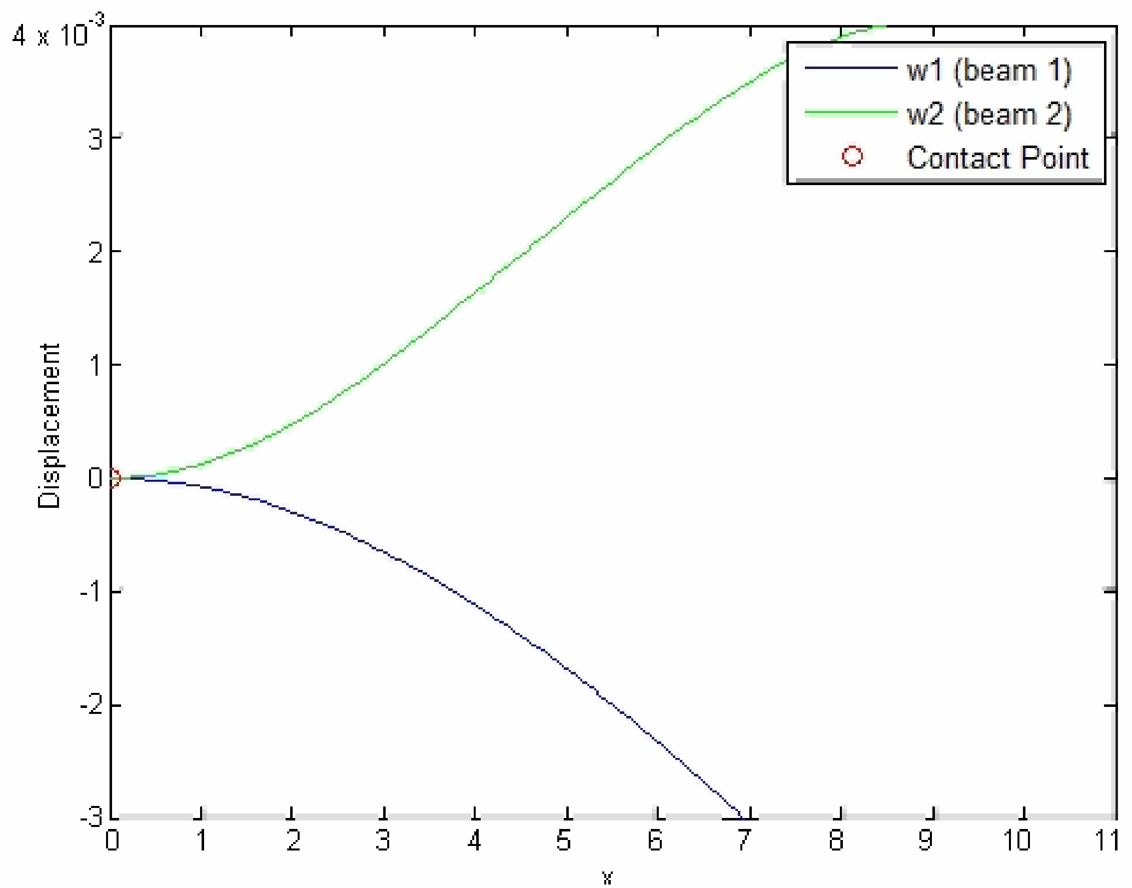
Comparison of Results

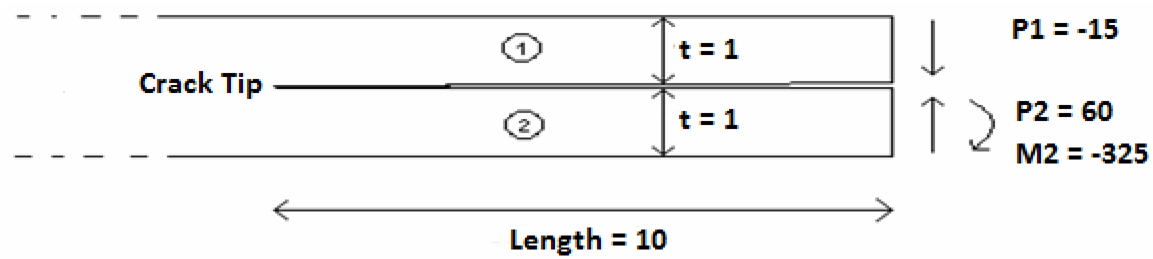
(Load case 2, $\Delta a/a$ = 1%)



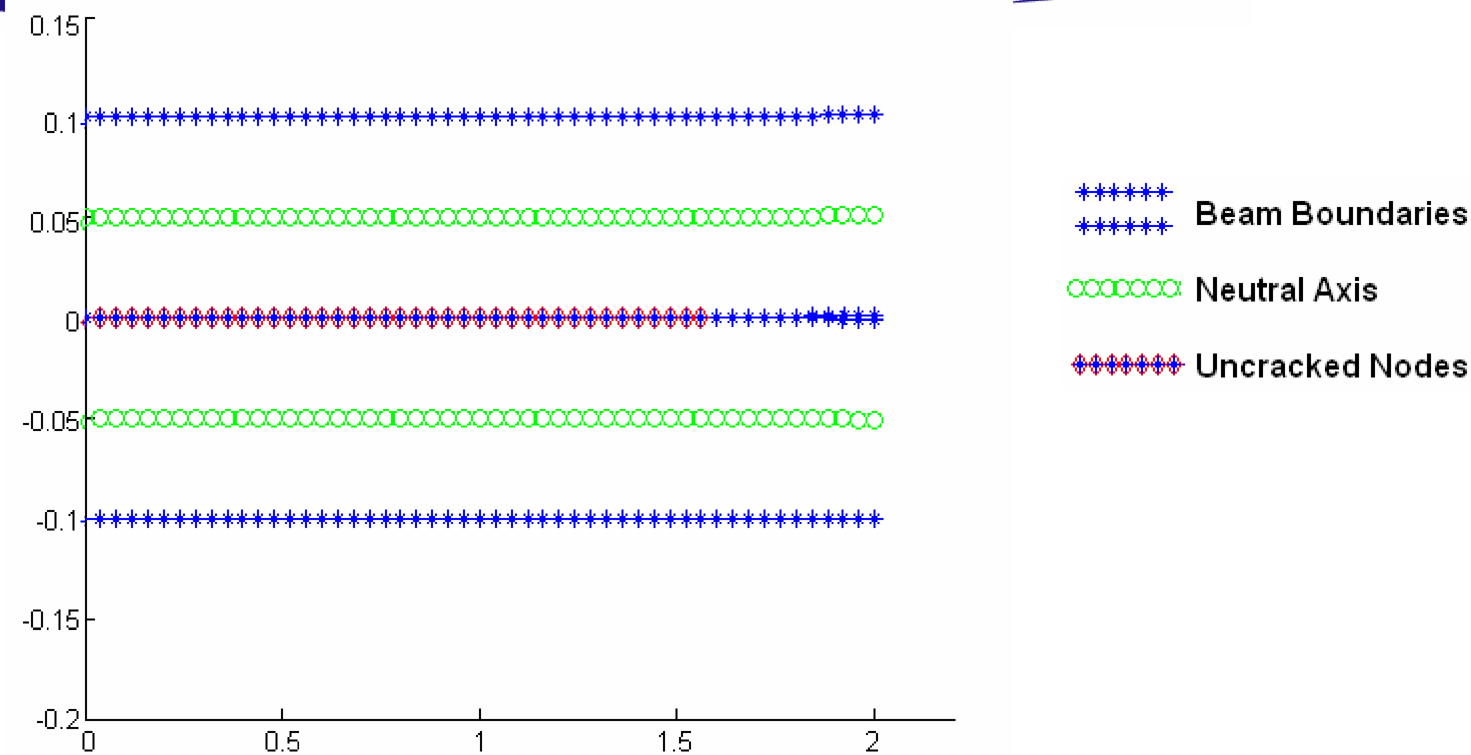


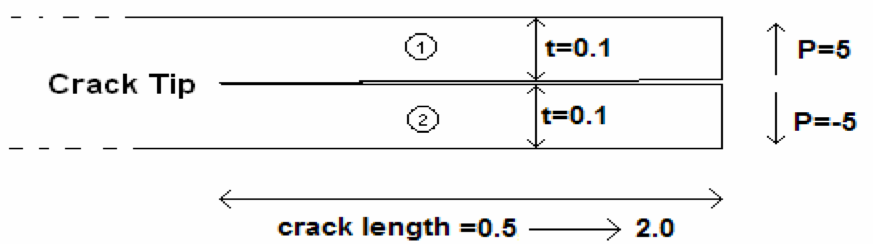
Resolving Contact





Crack Propagation



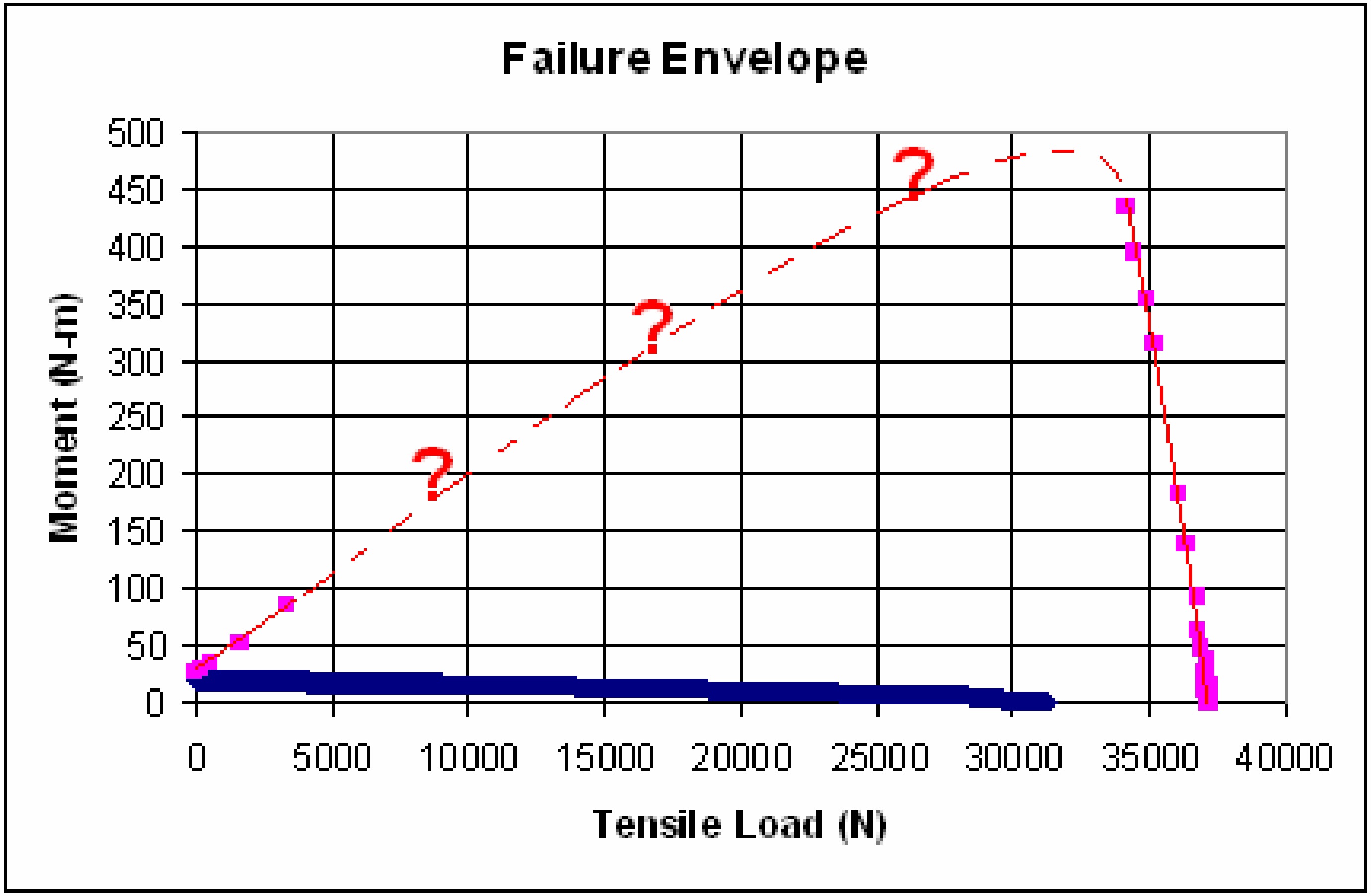


Work in Progress / Future Work

- Refine FEA models and procedures
- Develop analytical solutions
- Consider more failure modes
- Multiple fasteners
- Understand crack propagation around the fastener in 3-D
- Identify key variables for design and optimization
- Perform parametric/sensitivity analyses
- Design validation experiments

A Look Forward

- **Benefit to Aviation**
  - The present method allows engineers to design damage tolerant composite structures for a predetermined level of reliability, as required by FAR 25.
  - The present study makes it possible to determine the relationship among the reliability level, inspection interval, inspection method, and repair quality to minimize the maintenance cost and risk of structural failure.
- **Future needs**
  - A standardized methodology for establishing an optimal inspection schedule for aircraft manufacturers and operators.
  - Enhanced damage data reporting requirements regulated by the FAA.
  - A comprehensive system of characterizing variability of material properties.


Failure Envelope
Moment (N-m)
Tensile Load (N)
500
450
400
350
300
250
200
150
100
50
0
0
5000
10000
15000
20000
25000
30000
35000
40000
?
?
?